1 Equivalent theorem understanding in case of plane wave propagation

In case of plane wave propagation, the equivalent theorem is applied for a good understanding.

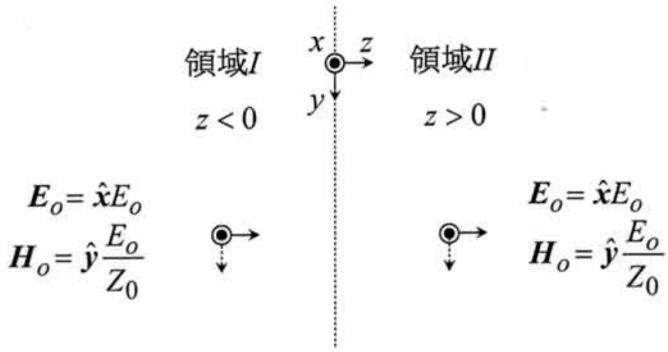

Figure 1: Original model

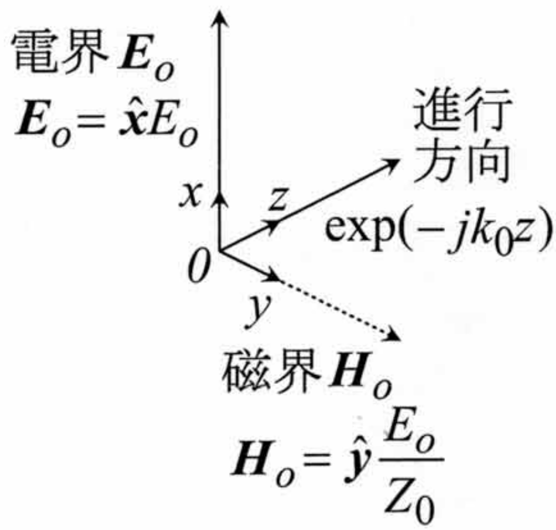

Figure 2: Plane wave

In fig.1, the free space is divided into two regions by an infinite plane at $z = 0$. For $z < 0$ and $z > 0$, the region I and region II are defined, respectively. For $+z$ propagation the plane wave is polarized in $+x$. In fig.2 the electromagnetic wave is shown and expressions are given as :

$$\mathbf{E}_o = \hat{\mathbf{x}} E_0 \exp\left(-jk_0 z\right) \tag{1}$$

$$\mathbf{H}_o = \hat{\mathbf{y}} \frac{E_0}{Z_0} \exp\left(-jk_0 z\right) \tag{2}$$

k_0 is wave number and Z_0 is impedance in free space.

1.1 Equivalent model for region II

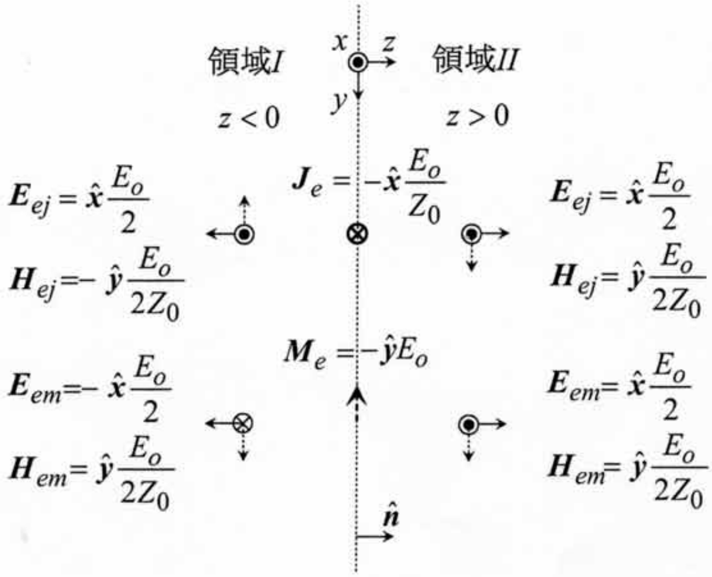

Figure 3: 領域 II の等価モデル (1)

In fig.3 the equivalent model for region II is shown. Because in region II plane wave does not exist, it is not taken into account (as real wave). We consider only the electromagnetic field produced by equivalent electromagnetic currents($\mathbf{J}_e$,$\mathbf{M}_e$) located on the boundary $S(z=0)$.

The equivalent electromagnetic currents($\mathbf{J}_e$,$\mathbf{M}_e$) on the boundary$S(z=0)$ are defined as:

$$\mathbf{J}_e = \hat{\mathbf{n}} \times \mathbf{H}_o = \hat{\mathbf{z}} \times \hat{\mathbf{y}}\frac{E_0}{Z_0} = -\hat{\mathbf{x}}\frac{E_0}{Z_0} \tag{3}$$

$$\mathbf{M}_e = \mathbf{E}_o \times \hat{\mathbf{n}} = \hat{\mathbf{x}} \times \hat{\mathbf{z}}E_0 = -\hat{\mathbf{y}}E_0 \tag{4}$$

where the vector $\hat{\mathbf{n}}$ is defined as unitary vector $\hat{\mathbf{z}}$ in the inner direction for region II.

The electromagnetic field ($\mathbf{E}_{ej}$,$\mathbf{H}_{ej}$) produced by equivalent electric current $\mathbf{J}_e$ is given as:

$$\mathbf{E}_{ej} = \begin{cases} \hat{\mathbf{x}}\frac{E_0}{2}\exp(jk_0z) & (z<0) \\ \hat{\mathbf{x}}\frac{E_0}{2}\exp(-jk_0z) & (z>0) \end{cases} \tag{5}$$

$$\mathbf{H}_{ej} = \begin{cases} -\hat{\mathbf{y}}\frac{E_0}{2Z_0}\exp(jk_0z) & (z<0) \\ \hat{\mathbf{y}}\frac{E_0}{2Z_0}\exp(-jk_0z) & (z>0) \end{cases} \tag{6}$$

where the magnetic field direction is given by clockwise rotation around the electric current. Because the propagation and magnetic field direction are known, the electric field direction is found by using right hand law. The plane wave produced by equivalent source propagates in $-z$ direction (region I) and $+z$ direction (region II).

The electromagnetic field ($\mathbf{E}_{em}$,$\mathbf{H}_{em}$) produced by equivalent electric current $\mathbf{M}_e$ is given as:

$$\mathbf{E}_{em} = \begin{cases} -\hat{\mathbf{x}}\frac{E_0}{2}\exp(jk_0z) & (z<0) \\ \hat{\mathbf{x}}\frac{E_0}{2}\exp(-jk_0z) & (z>0) \end{cases} \tag{7}$$

$$\mathbf{H}_{em} = \begin{cases} \hat{\mathbf{y}}\frac{E_0}{2Z_0}\exp(jk_0z) & (z<0) \\ \hat{\mathbf{y}}\frac{E_0}{2Z_0}\exp(-jk_0z) & (z>0) \end{cases} \tag{8}$$

where the electric field direction is given by counterclockwise rotation around the magnetic current. Because the propagation and magnetic field direction are known, the magnetic field direction is found by using right hand law.

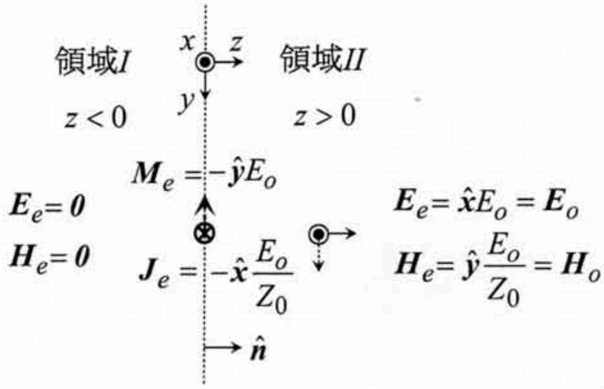

Figure 4: 領域IIの等価モデル(2)

Equivalent electromagnetic field($\mathbf{E}_e$,$\mathbf{H}_e$) is found adding contribution from electrical and magnetic equivalent field ($\mathbf{E}_{ej}$,$\mathbf{H}_{ej}$) and ($\mathbf{E}_{em}$,$\mathbf{H}_{em}$) according to the boundary.

$$\mathbf{E}_e = \mathbf{E}_{ej} + \mathbf{E}_{em} = \begin{cases} \mathbf{0} & (z<0) \\ \hat{\mathbf{x}}E_0\exp(-jk_0z) = \mathbf{E}_o & (z>0) \end{cases} \tag{9}$$

$$\mathbf{H}_e = \mathbf{H}_{ej} + \mathbf{H}_{em} = \begin{cases} \mathbf{0} & (z<0) \\ \hat{\mathbf{y}}\frac{E_0}{Z_0}\exp(-jk_0z) = \mathbf{H}_o & (z>0) \end{cases} \tag{10}$$

The same result is observed in fig.4. In region II original field is given and field becomes zero in region I.

1.2 Equivalent model for region I

In fig.5 the equivalent model for region I is shown. in region I the original plane wave ($\mathbf{E}_o$,$\mathbf{H}_o$) must be taken into account. in this region the original electromagnetic field and the field produced by the equivalent electromagnetic current must be added. because the vector n is defined as unitary vector in the inner direction for region I ($\hat{\mathbf{n}} = -\hat{\mathbf{z}}$), the equivalent electromagnetic currents on the boundary $S(z=0)$ become $-\mathbf{J}_e$ and $-\mathbf{M}_e$. From (9) and (10) the electromagnetic field ($\mathbf{E}_e$,$\mathbf{H}_e$) produced by these equivalent electromagnetic currents becomes zero in region I and minus the original region II.

$$\mathbf{E}_e = \begin{cases} \mathbf{0} & (z<0) \\ -\hat{\mathbf{x}}E_0\exp(-jk_0z) = -\mathbf{E}_o & (z>0) \end{cases} \tag{11}$$

$$\mathbf{H}_e = \begin{cases} \mathbf{0} & (z<0) \\ -\hat{\mathbf{y}}\frac{E_0}{Z_0}\exp(-jk_0z) = -\mathbf{H}_o & (z>0) \end{cases} \tag{12}$$

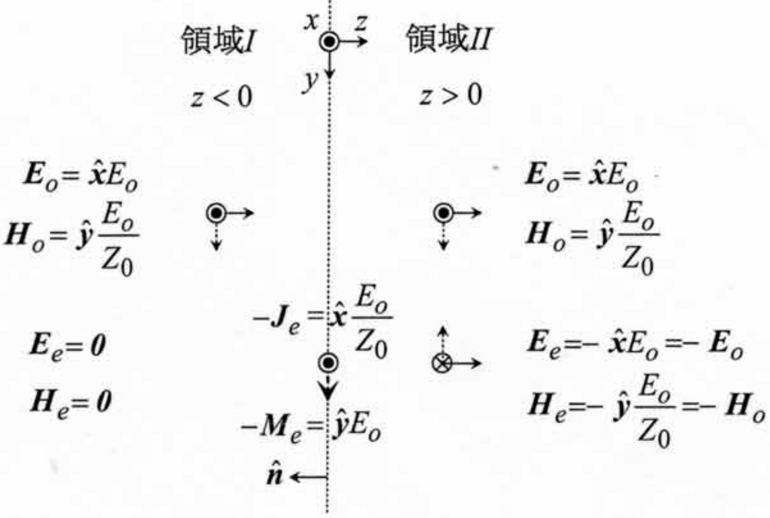

Figure 5: 領域 I の等価モデル (1)

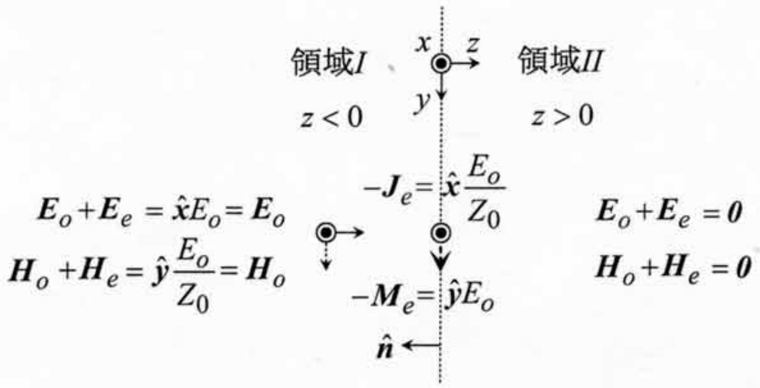

Figure 6: 領域 I の等価モデル (2)

Total electromagnetic field($\mathbf{E}_t$,$\mathbf{H}_t$) is found adding contribution from original electromagnetic field ($\mathbf{E}_o$,$\mathbf{H}_o$) and equivalent electromagnetic field ($\mathbf{E}_e$,$\mathbf{H}_e$).

$$\mathbf{E}_o + \mathbf{E}_e = \begin{cases} \hat{\mathbf{x}} E_0 \exp(-jk_0 z) = \mathbf{E}_o & (z < 0) \\ \mathbf{0} & (z > 0) \end{cases} \tag{13}$$

$$\mathbf{H}_o + \mathbf{H}_e = \begin{cases} \hat{\mathbf{y}} \frac{E_0}{Z_0} \exp(-jk_0 z) = \mathbf{H}_o & (z < 0) \\ \mathbf{0} & (z > 0) \end{cases} \tag{14}$$

The final result is observed in fig.6.

1.3 Equivalent model for region II when region I is perfect electric conductor (PEC)

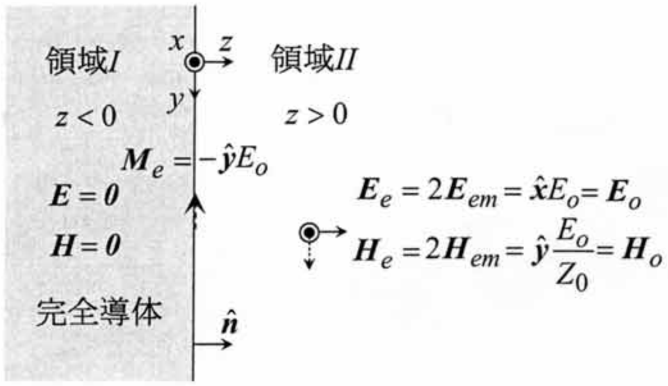

Figure 7: 領域 I を完全導体で満たした領域 II の等価モデル

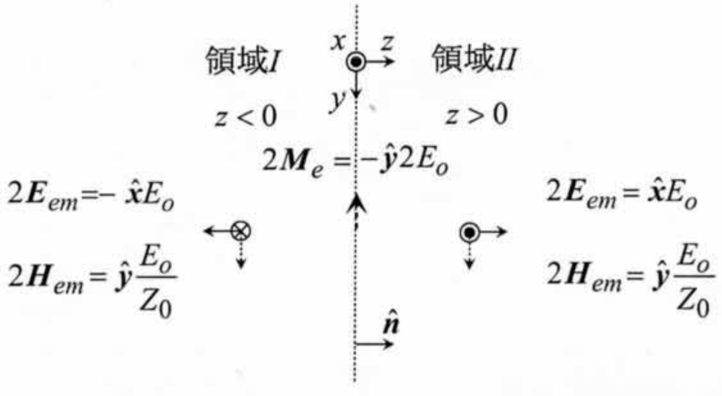

Figure 8: 等価電磁流 $2\mathbf{M}_e$ が自由空間に作る電磁界

Figure 7 shows case when region I is PEC. In this case the presence of electric wall forces existence of only equivalent magnetic current $\mathbf{M}_e$. When the electromagnetic field ($\mathbf{E}_e$,$\mathbf{H}_e$) produced by equivalent magnetic current is needed, the existence of electric wall S must be taken into account. Because the electric wall S is infinite plane, the electromagnetic field ($\mathbf{E}_e$,$\mathbf{H}_e$) produced by equivalent magnetic current Me for $z > 0$ might be calculated using image theory. when image theory is applied, region I is substituted by free space and the equivalent magnetic current becomes $2\mathbf{M}_e$. this argument is valid only for $z > 0$,where the equivalent electromagnetic field becomes $2\mathbf{E}_{em}$ and $2\mathbf{H}_{em}$ similar to the equations (7)(8).

$$\mathbf{E}_e = 2\mathbf{E}_{em} = \hat{\mathbf{x}}E_0 \exp(-jk_0 z) = \mathbf{E}_o \tag{15}$$

$$\mathbf{H}_e = 2\mathbf{H}_{em} = \hat{\mathbf{y}}\frac{E_0}{Z_0} \exp(-jk_0 z) = \mathbf{H}_o \tag{16}$$

Therefore, the original field appears in region II and is observed in fig.7. In region I the electromagnetic field vanishes because of PEC.

1.4 Equivalent model for region I when region II is perfect electric conductor (PEC)

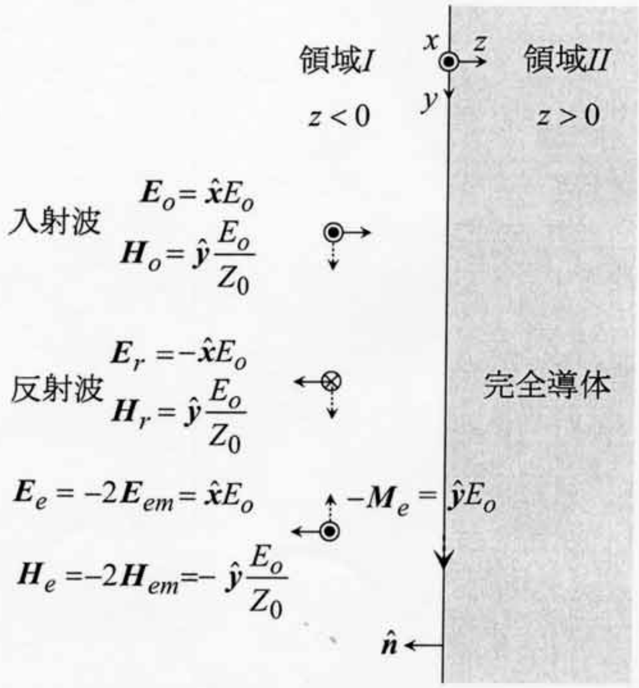

Figure 9: 領域 II を完全導体で満たした領域 I の等価モデル (1)

Figure 9 shows case when region II is PEC. In this case when the original field ($\mathbf{E}_o$,$\mathbf{H}_o$) hits the electric wall, the reflected field ($\mathbf{E}_r$,$\mathbf{H}_r$) is induced. On the electric wall the equivalent magnetic current $-\mathbf{M}_e$ produce the equivalent electromagnetic field ($\mathbf{E}_e$,$\mathbf{H}_e$). For the total contribution the three components are needed. The reflected wave ($\mathbf{E}_r$,$\mathbf{H}_r$) is defined so that the tangential components of electric field vanishes on the electric wall$S$.

$$\mathbf{E}_r = -\hat{\mathbf{x}}E_0 \exp(jk_0 z) \tag{17}$$

$$\mathbf{H}_r = \hat{\mathbf{y}}\frac{E_0}{Z_0} \exp(jk_0 z) \tag{18}$$

Because the electric wall S is infinite plane, the electromagnetic field ($\mathbf{E}_e$,$\mathbf{H}_e$) produced by equivalent magnetic current $-\mathbf{M}_e$ for $z < 0$ might be calculated using image theory. When

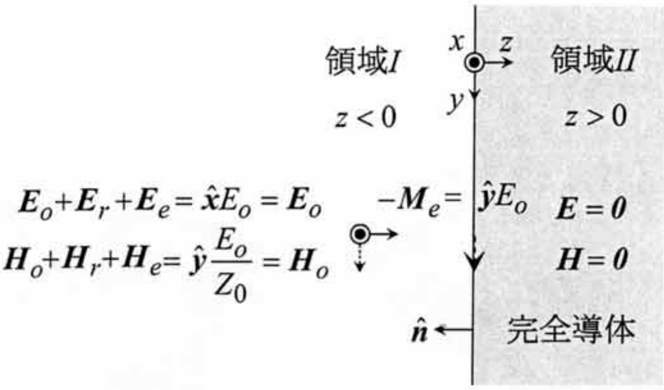

Figure 10: 領域 II を完全導体で満たした領域 I の等価モデル (2)

image theory is applied, region II is substituted by free space and the equivalent magnetic current becomes $-2\mathbf{M}_e$. This argument is valid only for $z < 0$,where the equivalent electromagnetic field becomes $-2\mathbf{E}_{em}$ and $-2\mathbf{H}_{em}$ similar to the equations (7)(8).

$$\mathbf{E}_e = \hat{\mathbf{x}} E_0 \exp{(jk_0 z)} \tag{19}$$

$$\mathbf{H}_e = -\hat{\mathbf{y}} \frac{E_0}{Z_0} \exp{(jk_0 z)} \tag{20}$$

then these three components must be added as:

$$\mathbf{E}_o + \mathbf{E}_r + \mathbf{E}_e = \hat{\mathbf{x}} E_0 \exp{(-jk_0 z)} = \mathbf{E}_o \tag{21}$$

$$\mathbf{H}_o + \mathbf{H}_r + \mathbf{H}_e = \hat{\mathbf{y}} \frac{E_0}{Z_0} \exp{(-jk_0 z)} = \mathbf{H}_o \tag{22}$$

therefore, finally in region I the original field is obtained. And in region II the electromagnetic field vanishes because of PEC.