Mechanical Vibration Ⅰ (15)

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Two-degrees-of-freedom Vibration System (2)

Modal Analysis (5)

Equation of motion with no excitation force

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}$$

Coordinate transformation

$$\mathbf{x} = \mathbf{V}\mathbf{y}$$

$$\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$$ Physical coordinate

$$\mathbf{y} = \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}$$ Modal coordinate

y_i : Displacement of the i-th mode

Modal Analysis (6)

Equation of motion with no excitation force

$$\mathbf{V}^T \mathbf{M} \mathbf{V} \ddot{\mathbf{y}} + \mathbf{V}^T \mathbf{K} \mathbf{V} \mathbf{y} = \mathbf{0}$$

$$\longrightarrow \quad \ddot{\mathbf{y}} + \Lambda \mathbf{y} = \mathbf{0}$$

where

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Asymmetric elements are 0.

Responses are divided into two independent modes.

Modal Analysis (7)

$$\ddot{y}_1 = 0 \longrightarrow y_1(t) = y_1(0) + \dot{y}_1(0)t$$

$$\ddot{y}_2 + \lambda_2 y_2 = 0$$

$$\longrightarrow y_2(t) = y_2(0)\cos(\omega_2 t) + \frac{\dot{y}_2(0)}{\omega_2}\sin(\omega_2 t)$$

where

$$\omega_2 = \sqrt{\lambda_2} = \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}}$$

Modal Analysis (8)

Initial conditions

$$\begin{Bmatrix} y_1(0) \\ y_2(0) \end{Bmatrix} = \mathbf{V}^{-1} \begin{Bmatrix} x_1(0) \\ x_2(0) \end{Bmatrix}, \quad \begin{Bmatrix} \dot{y}_1(0) \\ \dot{y}_2(0) \end{Bmatrix} = \mathbf{V}^{-1} \begin{Bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \end{Bmatrix}$$

Reconstruction of Physical coordinates

$$\mathbf{x}(t) = \mathbf{V}\mathbf{y}(t)$$

Modal Analysis (9)

Forced response

Equation of motion with excitation force

$$\mathbf{V}^T\mathbf{M}\mathbf{V}\ddot{\mathbf{y}} + \mathbf{V}^T\mathbf{K}\mathbf{V}\mathbf{y} = \mathbf{V}^T\mathbf{f}$$

$$\rightarrow \quad \ddot{\mathbf{y}} + \Lambda\mathbf{y} = \mathbf{f}_m$$

where

$$\mathbf{f}_m = \begin{Bmatrix} \mathbf{v}_1{}^T\mathbf{f} \\ \mathbf{v}_2{}^T\mathbf{f} \end{Bmatrix} = \begin{Bmatrix} f_{m1} \\ f_{m2} \end{Bmatrix}$$

Modal Analysis (10)

$$\ddot{y}_1 = f_{m1} \longrightarrow y_1(t) = y_1(0) + \dot{y}_1(0)t + \int_0^t (t-\tau) f_{m1}(\tau)\, d\tau$$

$$\ddot{y}_2 + \lambda_2 y_2 = f_{m2} \longrightarrow y_2(t) = y_2(0)\cos(\omega_2 t) + \frac{\dot{y}_2(0)}{\omega_2}\sin(\omega_2 t) + \frac{1}{\omega_2}\int_0^t \sin\{\omega_2(t-\tau)\} f_{m2}(\tau)\, d\tau$$