Mechanical Vibration Ⅰ (11)

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Response for an Arbitrary Exciting Force (1)

Response for a harmonic excitation with one frequency component

Complex Excitation Force

Response

$$f(t) = F_1 \exp(i\omega_1 t) \longrightarrow x(t) = F_1 H(i\omega_1 t)\exp(i\omega_1 t)$$

Harmonic Excitation Force with one frequency component

$$f(t) = F_1 \cos(\omega_1 t) = F_1 \operatorname{Re}\{\exp(i\omega_1 t)\}$$

$$\Downarrow$$

$$x(t) = F_1 \operatorname{Re}\{H(i\omega_1)\exp(i\omega_1 t)\}$$

$$= F_1\left[\operatorname{Re}\{H(i\omega_1)\}\cos(\omega_1 t) - \operatorname{Im}\{H(i\omega_1)\}\sin(\omega_1 t)\right]$$

Response for a harmonic excitation with two frequency component

Harmonic Excitation Force with two frequency component

$$f(t) = F_1\cos(\omega_1 t) + F_2\sin(\omega_2 t)$$

$$\Downarrow$$

$$x(t) = F_1\,\mathrm{Re}\{H(i\omega_1)\exp(i\omega_1 t)\} + F_2\,\mathrm{Im}\{H(i\omega_2)\exp(i\omega_2 t)\}$$

$$= F_1\left[\mathrm{Re}\{H(i\omega_1)\}\cos(\omega_1 t) - \mathrm{Im}\{H(i\omega_1)\}\sin(\omega_1 t)\right]$$

$$+ F_2\left[\mathrm{Re}\{H(i\omega_2)\}\sin(\omega_2 t) + \mathrm{Im}\{H(i\omega_2)\}\cos(\omega_2 t)\right]$$

Response for a periodic excitation with a period of *T* (1)

Periodic Excitation Force with a period of *T*

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2n\pi}{T}t\right) + b_n \sin\left(\frac{2n\pi}{T}t\right)\right]$$

where

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} f(t)dt,\ a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\left(\frac{2n\pi}{T}t\right)dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin\left(\frac{2n\pi}{T}t\right)dt$$

Response for a periodic excitation with a period of *T* (2)

Response for a periodic excitation (1)

$$x(t) = \frac{a_0}{2} H(0)$$

$$+ \sum_{n=1}^{\infty} \left[a_n \,\mathrm{Re}\left\{ H\left(i\frac{2n\pi}{T} \right) \right\} + b_n \,\mathrm{Im}\left\{ H\left(i\frac{2n\pi}{T} \right) \right\} \right] \cos\left(\frac{2n\pi}{T} t \right)$$

$$+ \left[-a_n \,\mathrm{Im}\left\{ H\left(i\frac{2n\pi}{T} \right) \right\} + b_n \,\mathrm{Re}\left\{ H\left(i\frac{2n\pi}{T} \right) \right\} \right] \sin\left(\frac{2n\pi}{T} t \right)$$

Response for a periodic excitation with a period of *T* (3)

Response for a periodic excitation (2)

$$x(t) = \sum_{n=-\infty}^{\infty} c_n H\left(i\frac{2n\pi}{T} \right) \exp\left(i\frac{2n\pi}{T} t \right)$$

where

$$c_0 = \frac{a_0}{2}, \ c_n = \begin{cases} \dfrac{a_n - ib_n}{2}, & n > 0 \\ \dfrac{a_{-n} + ib_{-n}}{2}, & n < 0 \end{cases}$$