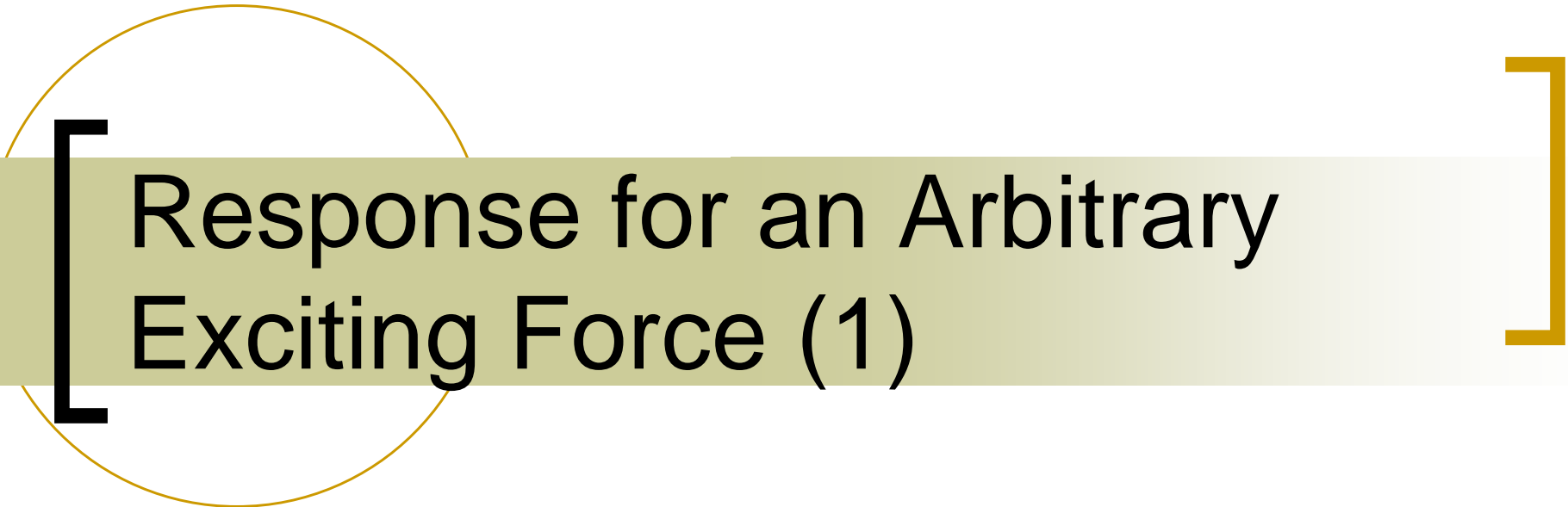


A decorative graphic consisting of a thin yellow circle and a horizontal bar. The bar has a yellow-to-white gradient and is flanked by large black and yellow brackets. The title text is centered within the yellow portion of the bar.

Mechanical Vibration (11)

Department of Mechanical and
Control Engineering

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Response for an Arbitrary Exciting Force (1)

Response for a harmonic excitation with one frequency component

Complex Excitation Force

Response

$$f(t) = F_1 \exp(i\omega_1 t) \quad \longrightarrow \quad x(t) = F_1 H(i\omega_1 t) \exp(i\omega_1 t)$$

Harmonic Excitation Force with one frequency component

$$f(t) = F_1 \cos(\omega_1 t) = F_1 \operatorname{Re}\{\exp(i\omega_1 t)\}$$



$$x(t) = F_1 \operatorname{Re}\{H(i\omega_1) \exp(i\omega_1 t)\}$$

$$= F_1 [\operatorname{Re}\{H(i\omega_1)\} \cos(\omega_1 t) - \operatorname{Im}\{H(i\omega_1)\} \sin(\omega_1 t)]$$

Response for a harmonic excitation with two frequency component

Harmonic Excitation Force with two frequency component

$$f(t) = F_1 \cos(\omega_1 t) + F_2 \sin(\omega_2 t)$$



$$\begin{aligned} x(t) &= F_1 \operatorname{Re}\{H(i\omega_1)\exp(i\omega_1 t)\} + F_2 \operatorname{Im}\{H(i\omega_2)\exp(i\omega_2 t)\} \\ &= F_1 [\operatorname{Re}\{H(i\omega_1)\} \cos(\omega_1 t) - \operatorname{Im}\{H(i\omega_1)\} \sin(\omega_1 t)] \\ &\quad + F_2 [\operatorname{Re}\{H(i\omega_2)\} \sin(\omega_2 t) + \operatorname{Im}\{H(i\omega_2)\} \cos(\omega_2 t)] \end{aligned}$$

Response for a periodic excitation with a period of T (1)

Periodic Excitation Force with a period of T

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{2n\pi}{T}t\right) + b_n \sin\left(\frac{2n\pi}{T}t\right) \right]$$

where

$$a_0 = \frac{2}{T} \int_{-T/2}^{T/2} f(t) dt, \quad a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos\left(\frac{2n\pi}{T}t\right) dt$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin\left(\frac{2n\pi}{T}t\right) dt$$

Response for a periodic excitation with a period of T (2)

Response for a periodic excitation (1)

$$\begin{aligned} x(t) = & \frac{a_0}{2} H(0) \\ & + \sum_{n=1}^{\infty} \left[a_n \operatorname{Re} \left\{ H \left(i \frac{2n\pi}{T} \right) \right\} + b_n \operatorname{Im} \left\{ H \left(i \frac{2n\pi}{T} \right) \right\} \right] \cos \left(\frac{2n\pi}{T} t \right) \\ & + \left[-a_n \operatorname{Im} \left\{ H \left(i \frac{2n\pi}{T} \right) \right\} + b_n \operatorname{Re} \left\{ H \left(i \frac{2n\pi}{T} \right) \right\} \right] \sin \left(\frac{2n\pi}{T} t \right) \end{aligned}$$

Response for a periodic excitation with a period of T (3)

Response for a periodic excitation (2)

$$x(t) = \sum_{n=-\infty}^{\infty} c_n H\left(i \frac{2n\pi}{T}\right) \exp\left(i \frac{2n\pi}{T} t\right)$$

where

$$c_0 = \frac{a_0}{2}, \quad c_n = \begin{cases} \frac{a_n - ib_n}{2}, & n > 0 \\ \frac{a_{-n} + ib_{-n}}{2}, & n < 0 \end{cases}$$