Mechanical Vibration Ⅰ (8)

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Complex Exciting Force

Complex exciting force and response (1)

Damped one degree-of-freedom system

$$\boxed{m\ddot{x} + c\dot{x} + kx = f}$$

Exciting force

Response

$$f_c = F\cos\omega t \quad \Rightarrow \quad x(t) = A\cos(\omega t + \phi)$$

$$f_s = F\sin\omega t \quad \Rightarrow \quad x(t) = A\sin(\omega t + \phi)$$

Complex exciting force and response (2)

Complex exciting force

$$f(t) = F\cos\omega t + iF\sin\omega t$$
$$= F\exp(i\omega t)$$

Response

$$\begin{aligned} x(t) &= A\cos(\omega t + \phi) + iA\sin(\omega t + \phi) \\ &= A\exp(i\phi)\exp(i\omega t) \\ &= X(i\omega)\exp(i\omega t) \end{aligned}$$

Complex exciting force and response (3)

If once $X(i\boldsymbol{\omega})$ is calculated,

$$x_c(t) = \mathrm{Re}\{X(i\omega)\exp(i\omega t)\}$$

$$x_s(t) = \mathrm{Im}\{X(i\omega)\exp(i\omega t)\}$$

and

$$A(\omega) = |X(i\omega)|$$

$$\phi(\omega) = \angle X(i\omega)$$

Frequency response function for force excitation (1)

Damped one degree-of-freedom system

$$\boxed{m\ddot{x} + c\dot{x} + kx = f}$$

Assume $x(t) = X(i\omega)\exp(i\omega t)$

and $f(t) = F\exp(i\omega t)$

then

$$\boxed{\frac{X(i\omega)}{F} = \frac{1}{(k - m\omega^2) + ic\omega}}$$

Frequency response function for force excitation (2)

Define $$X_{st} = \frac{F}{k}$$

Dynamic amplitude ratio

$$\boxed{\frac{X(i\omega)}{X_{st}} = \frac{k}{(k - m\omega^2) + ic\omega} = \frac{1}{(1-\Omega^2) + i2\zeta\Omega}}$$

where $$\Omega = \frac{\omega}{\omega_n}$$

Frequency response function for force excitation (3)

$$Gain(\mathrm{dB}) = 20\log_{10}\left|\frac{X(i\omega)}{X_{st}}\right|$$

$$= 20\log_{10}\left|\frac{1}{(1-\Omega^2)+i2\zeta\Omega}\right|$$

$$Phase(\mathrm{deg}) = \tan^{-1}\left(\frac{-2\zeta\Omega}{1-\Omega^2}\right)$$

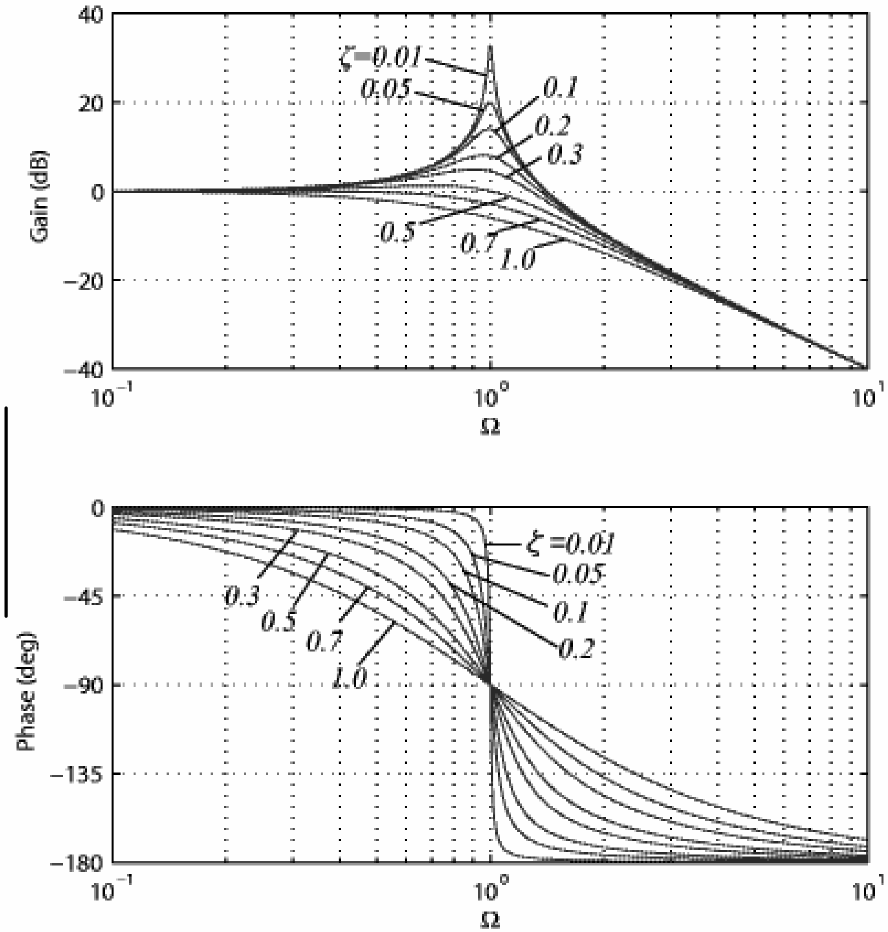

Fig.1 Frequency response function of the dynamic amplitude ratio