Mechanical Vibration Ⅰ (7)

Department of Mechanical and Control Engineering

Hiroshi Yamaura

Damped Vibration System (2) and Harmonic Excitation Response

Identification of system parameter (1)

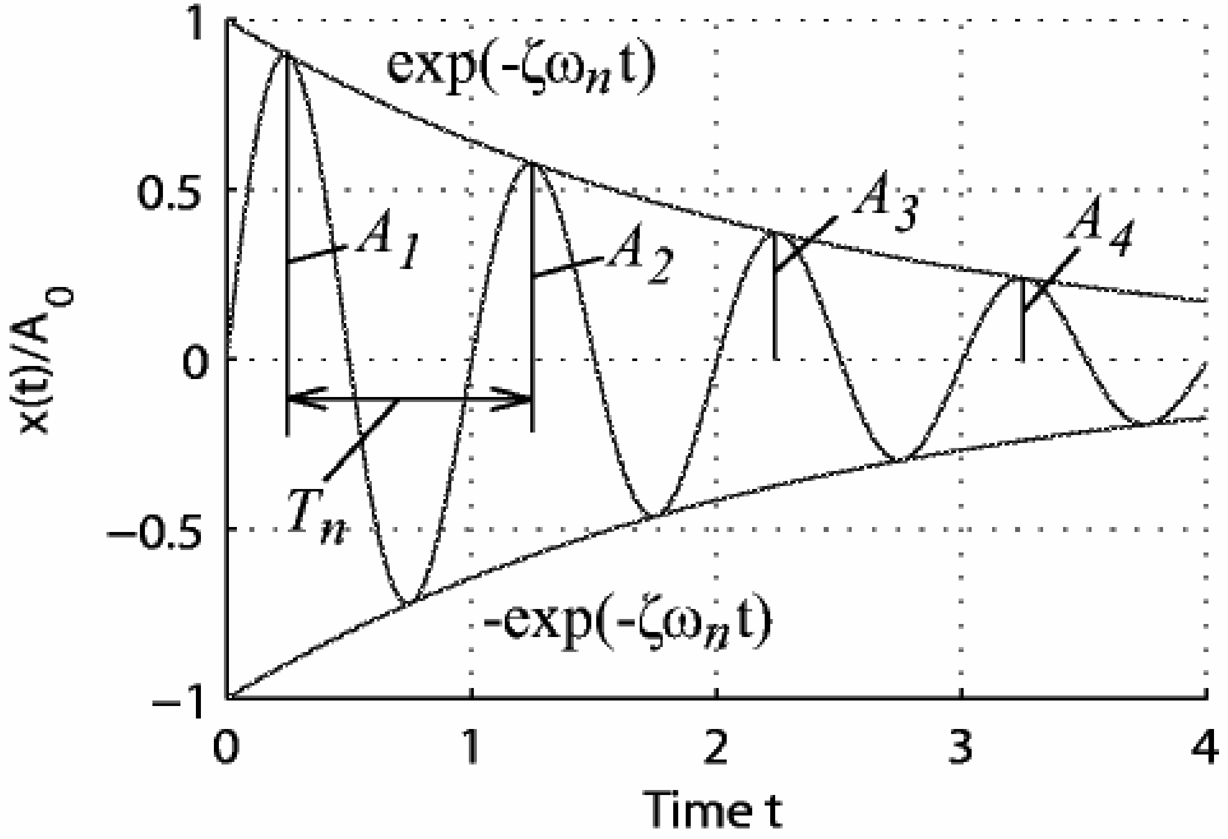

Fig.5 Free vibration of an actual vibration system to be modeled

Identification of system parameter (2)

Natural period T_n

Logarithmic damping ratio $$\delta = \frac{1}{N}\sum_{i=1}^{N}\ln\left(\frac{A_i}{A_{i+1}}\right)$$

Damping ratio $$\zeta = \frac{\delta}{2\pi\sqrt{1+\left(\frac{\delta}{2\pi}\right)^2}}$$

Identification of system parameter (3)

Case 1) *m* can be measured $k = m\omega_n^2 \quad c = 2\sqrt{mk}\zeta$

Case 2) *k* can be measured $m = \dfrac{k}{\omega_n^2} \quad c = 2\sqrt{mk}\zeta$

Harmonic exciting force and response (1)

Undamped one degree-of-freedom system

$$m\ddot{x} + kx = f$$

Exciting force

$$f_c = F\cos\omega t$$

$$A = \frac{F}{k - m\omega^2}$$

Response

$$x(t) = A\cos(\omega t + \phi)$$

$$\phi = \begin{cases} 0 & 0 \le \omega < \omega_n \\ -\pi/2 & \omega = \omega_n \\ -\pi & \omega > \omega_n \end{cases}$$

Harmonic exciting force and response (2)

Damped one degree-of-freedom system

$$m\ddot{x} + c\dot{x} + kx = f$$

Exciting force

$$f_c = F\cos\omega t$$

Response

$$x(t) = A\cos(\omega t - \phi)$$

$$A = \frac{F}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$\phi = \tan^{-1}\left(\frac{-c\omega}{k - m\omega^2}\right)$$