Mechanical Vibration Ⅰ (5)

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Energy method and Rayliegh's method

Energy method (1)

Undamped one degree-of-freedom system

$$m\ddot{x} + kx = 0$$ Conservative System

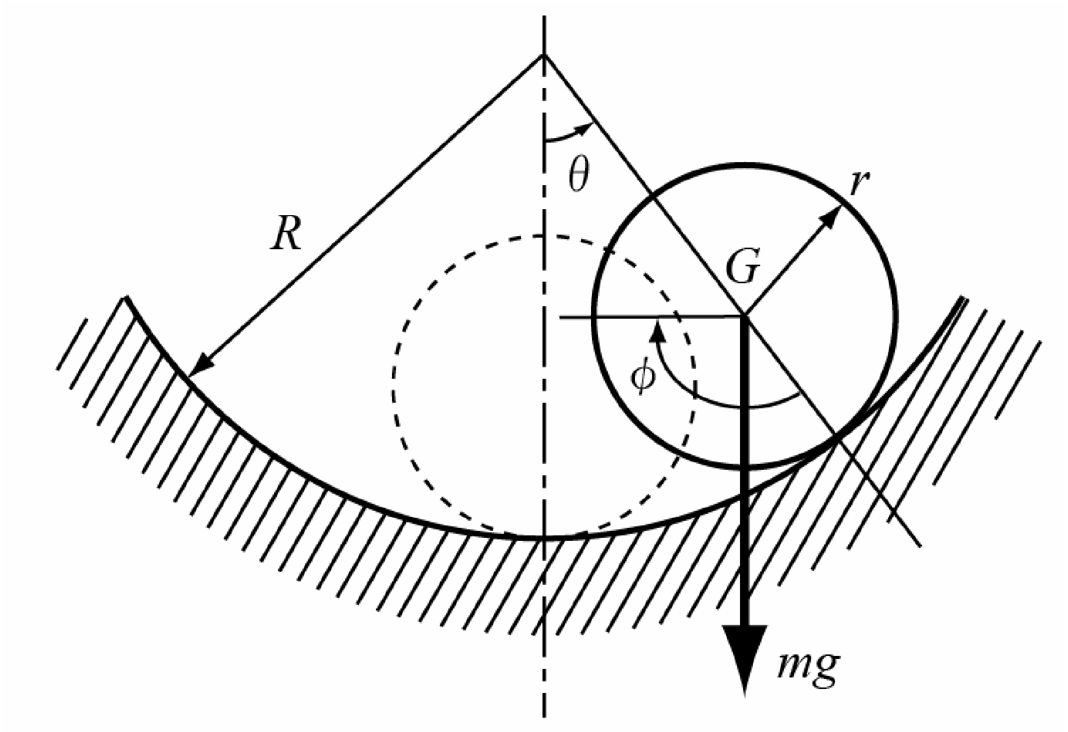
Mechanical Energy $E = T + U = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$

The mechanical energy of a conservative system is invariant during motion.

$$\Rightarrow \quad \frac{dE}{dt} = 0$$

Energy method (2)

Example：Rolling vibration



Rayliegh's method (1)

The mechanical energy of a conservative system is invariant during motion.

$$E = Const.$$

$$T_{\max} = U_{\max}$$

Rayliegh's method (2)

(1) Assume the shape of the vibration that is often called the mode shape.

(2) Calculate the maximum value of the kinetic energy and that of the potential energy of the vibration system.

(3) From the equivalence of the maximum value of the kinetic energy and that of the potential energy, the natural angular frequency is calculated.

Rayliegh's method (3)

Example 1：Linear Spring

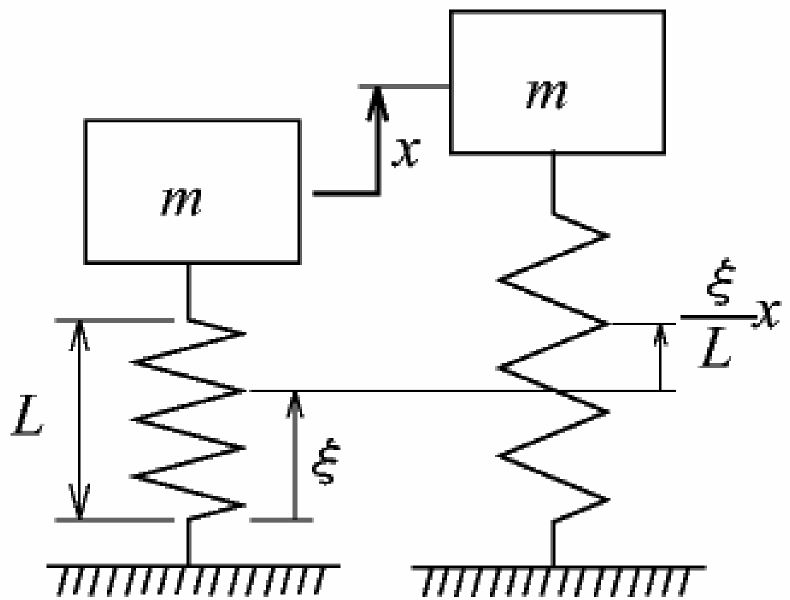


Fig.1 Effect of the mass of the spring on the natural angular frequency

Rayliegh's method (4)

Example 2：Mass-Cantilever System

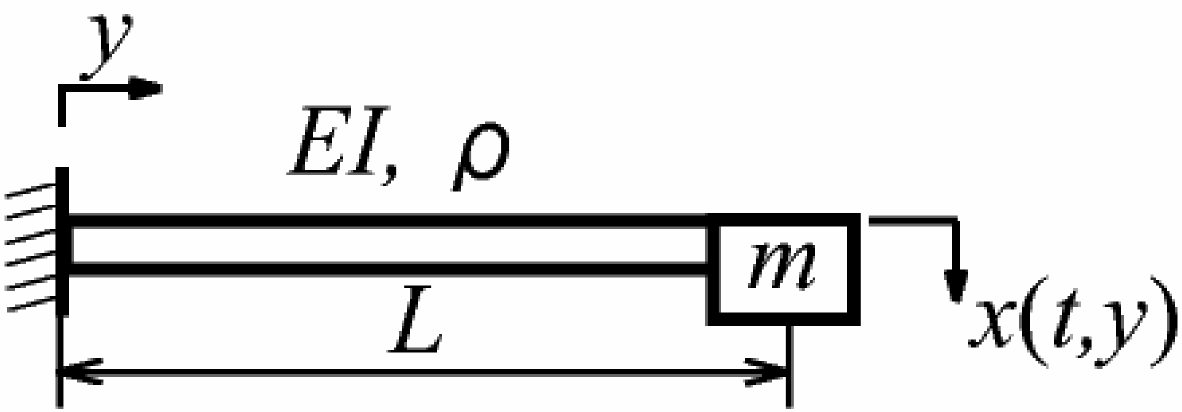


Fig.2 Effect of the mass of the beam on the natural angular frequency