Mechanical Vibration I (2)

Department of Mechanical and Control Engineering

Hiroshi Yamaura

Undamped One-degree-of-freedom Vibration System

Natural frequency

Free vibration

Natural angular frequency

Equation of Motion of an undamped and free vibration system

$$m\ddot{x} + kx = 0$$

The characteristic equation

$$m\lambda^2 + k = 0 \quad \Rightarrow \quad \lambda = \pm i\,\omega_n, \quad \omega_n = \sqrt{\frac{k}{m}}$$

Free vibration – Initial value response (1)

General solution

$$x(t) = C_1 e^{i\omega_n t} + C_2 e^{-i\omega_n t}$$

$$= D_1 \cos \omega_n t + D_2 \sin \omega_n t$$

$$= A \cos (\omega_n t - \phi)$$

Free vibration – Initial value response (2)

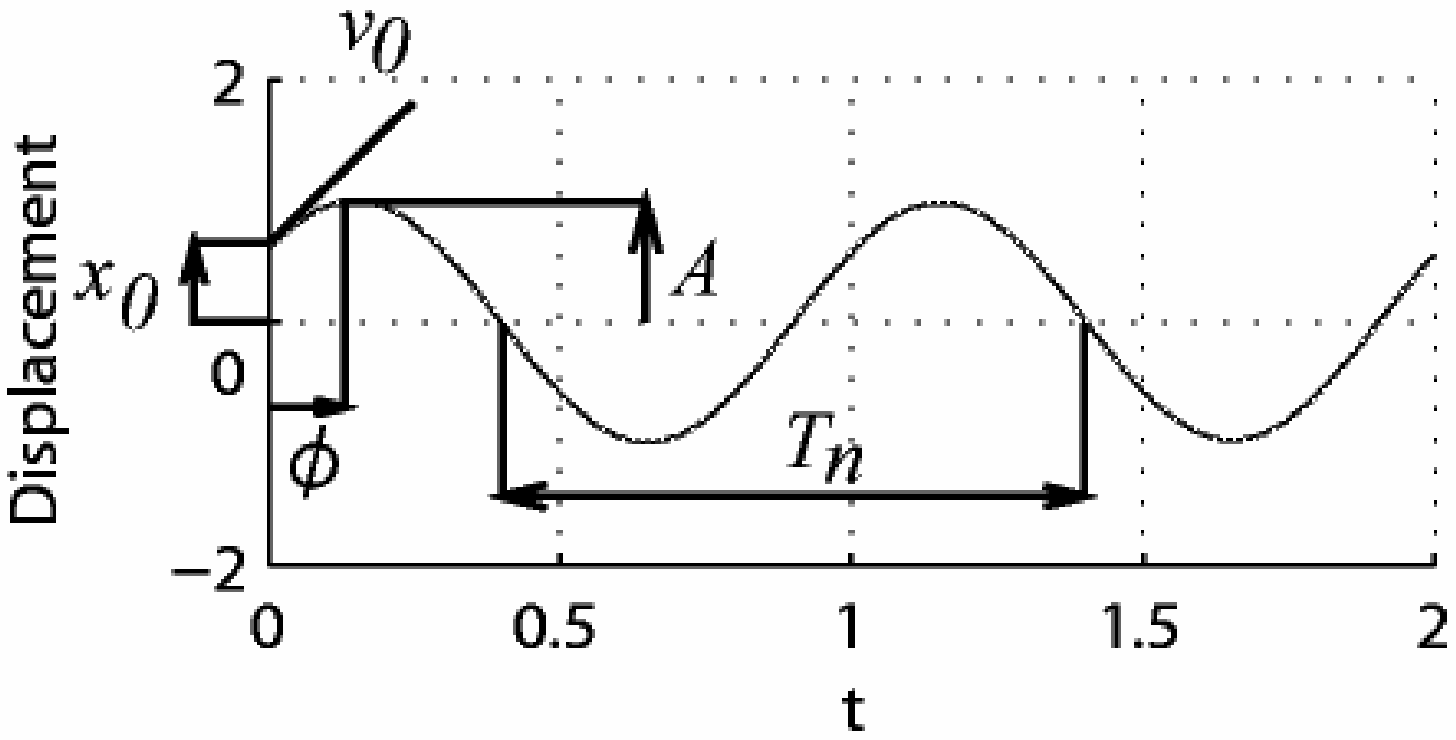

Fig.6 Example of initial value response

Natural angular frequency, natural frequency and natural period

Table 2 Important parameters

Notation	Unit	Meaning	
ω_n	rad/s	Natural angular frequency	$= \sqrt{\frac{k}{m}}$
f_n	Hz (=1/s)	Natural frequency	$= \frac{\omega_n}{2\pi}$
T_n	s	Natural period	$= \frac{1}{f_n} = \frac{2\pi}{\omega_n}$