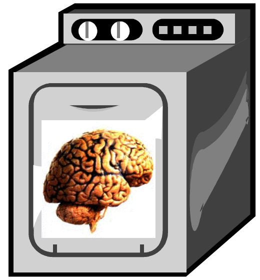
Pattern Information Processing
パターン情報処理

Masashi Sugiyama
(Department of Computer Science)
杉山　将（計算工学専攻）

Contact:　W8E-505
sugi@cs.titech.ac.jp
http://sugiyama-www.cs.titech.ac.jp/~sugi/

3 Topics in Learning Research

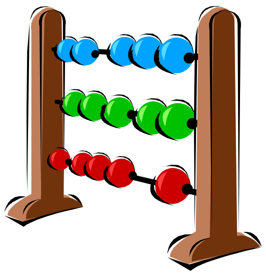


Understanding the brain

(physiology, psychology, neuroscience)

Developing learning machines

(computer and electronic engineering)

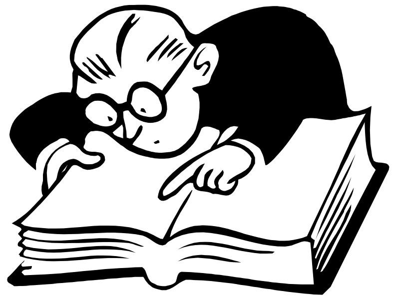


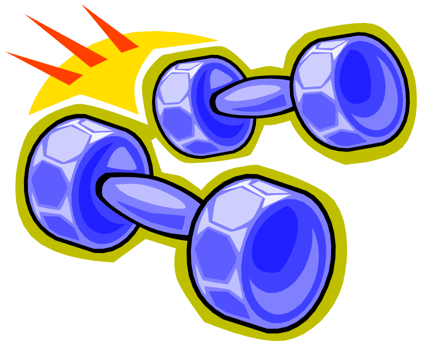
Clarifying learning mathematically

(computer and information science)

Three Types of Learning

- **Supervised learning:**
  Estimating underlying rule with supervisor's help.
- **Unsupervised learning:**
  Finding meaningful structure in data without supervisor.
- **Reinforcement learning:**
  Estimating underlying rule without supervisor.





Small Reports

- Write your original applications of
  - supervised learning,
  - unsupervised learning,
  - reinforcement learning.
- Relating to your own research topics would be a good idea.

- Deadline: Nov. 12 (Fri)

Today's Plan

- Supervised learning
- 3 important topics in supervised learning
  - Active learning
  - Model selection
  - Learning method
- Generalization error

Supervised Learning

- Supervisor has some knowledge.
- We (or a learning machine) want to learn supervisor’s knowledge.
- We can not directly access to supervisor’s knowledge.
- We are allowed to ask questions to the supervisor and he answers your questions.

Supervised Learning (cont.)

- Through pairs of questions and answers, we want to acquire the entire knowledge of our supervisor.
- Then, we can answer to the questions that we have never learned, i.e., we have the generalization capability.

Regression/Classification

- Suppose questions are real values.
- When the answers are
  - Discrete: Classification
  - Continuous: Regression
- We focus on regression.

Regression as Function Approximation

- Regression can be regarded as a function approximation problem.
- There is an unknown function $f(\boldsymbol{x})$.
- We are given its samples $\{(\boldsymbol{x}_i, y_i)\}_{i=1}^n$, where output values are generally noisy:

$$y_i = f(\boldsymbol{x}_i) + \epsilon_i$$

- We want to estimate $f(\boldsymbol{x})$.
- $\widehat{f}(\boldsymbol{x})$: Learned function

Regression as Function Approximation (cont.)

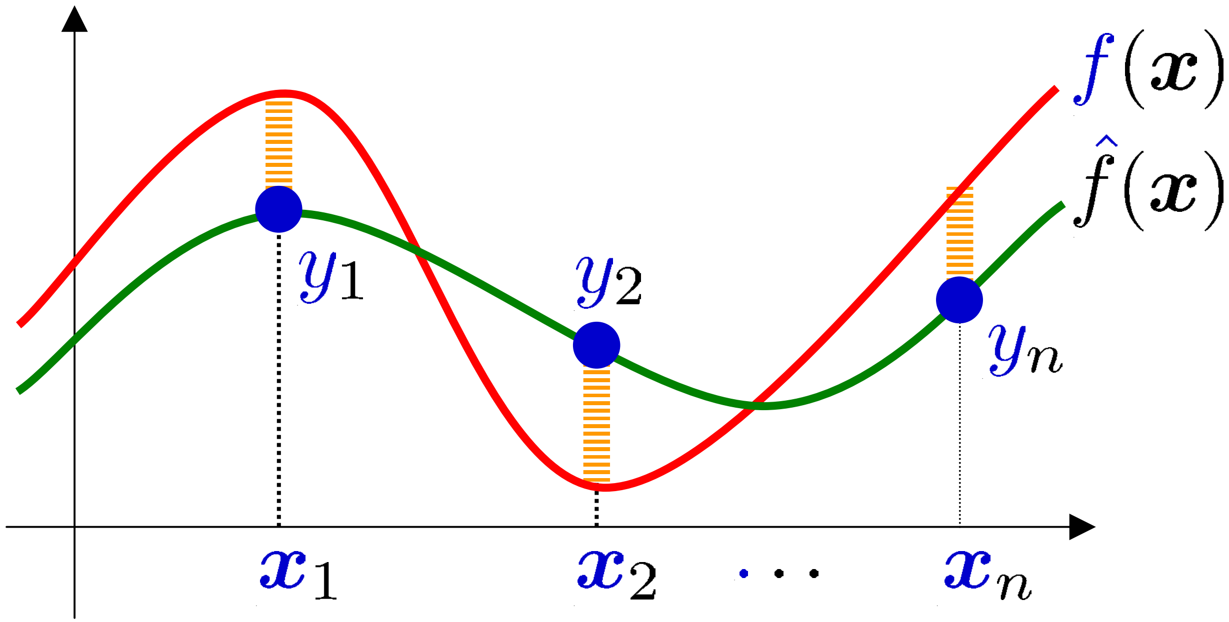
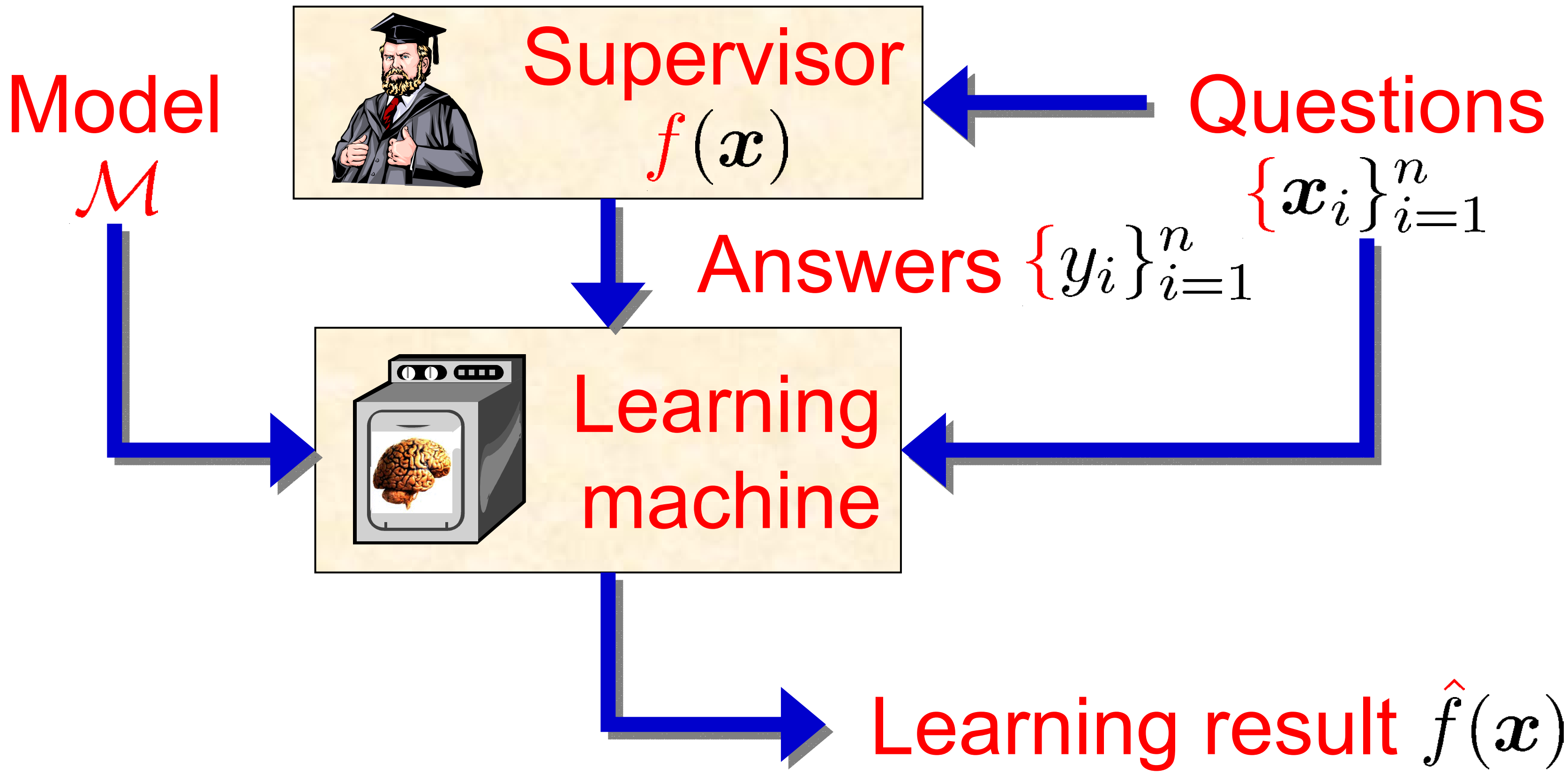


Diagram of Supervised Learning



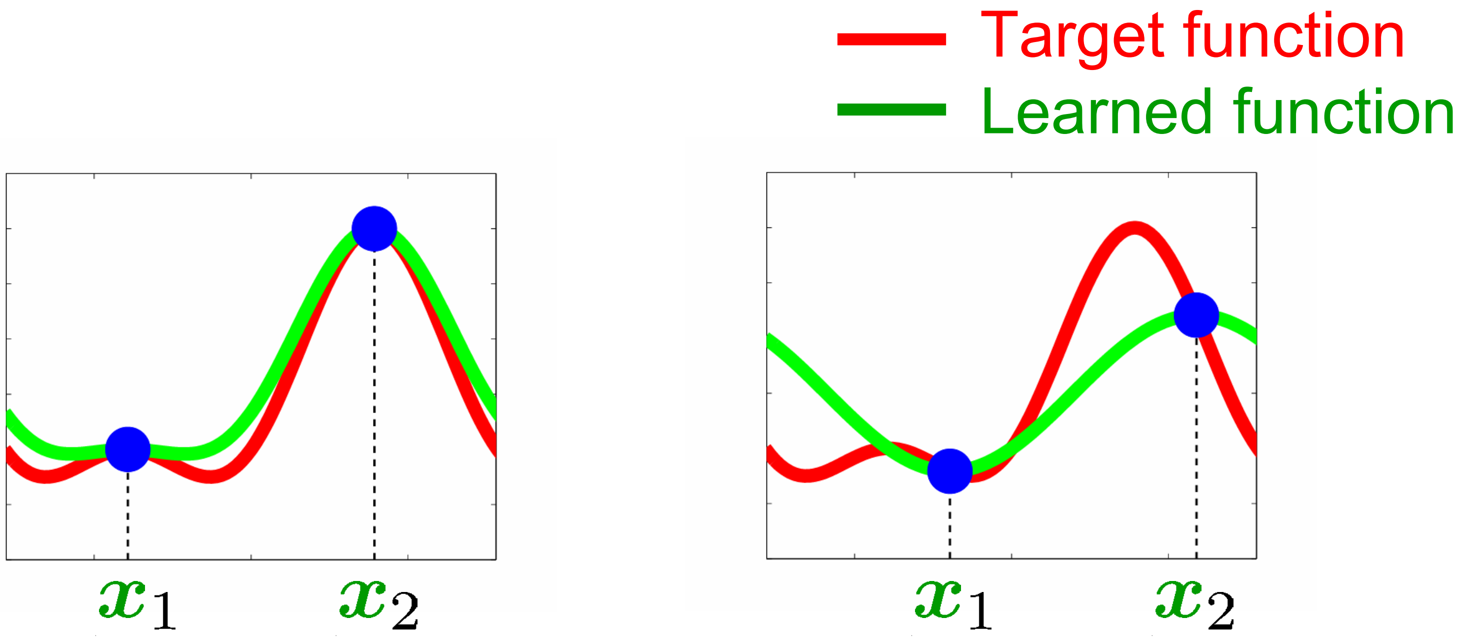
Model is a set of functions
from which $\widehat{f}(\boldsymbol{x})$ is searched.

3 Important Topics in Supervised Learning

- Active learning:
  What are the best questions to ask?
- Model selection:
  What is the best model to use?
- Learning method:
  What is the best way to learn?

Active Learning

For obtaining good learning results, questions should be determined appropriately.

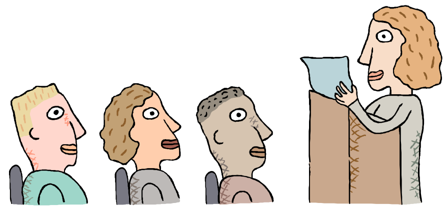


Good questions

Bad questions

Active Learning: Analogy to Real Life

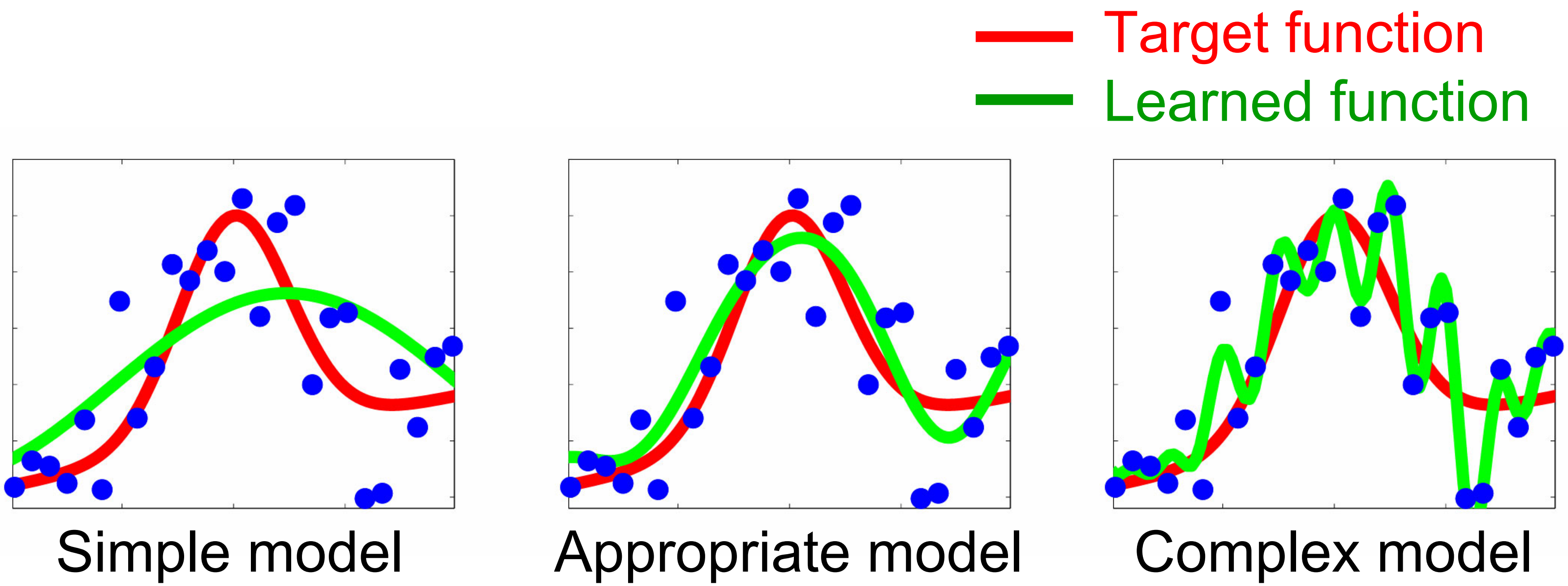
- It is not effective to passively attend the course.



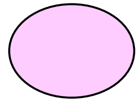
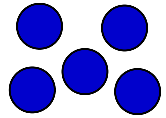
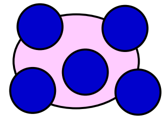
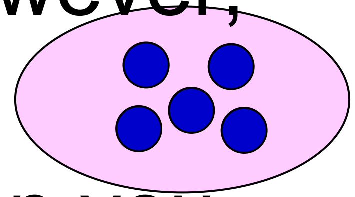
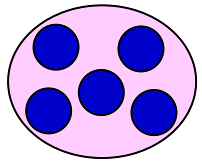
- Actively asking questions would be more effective for learning.

Model Selection

For obtaining good learning results,
model should be determined appropriately.

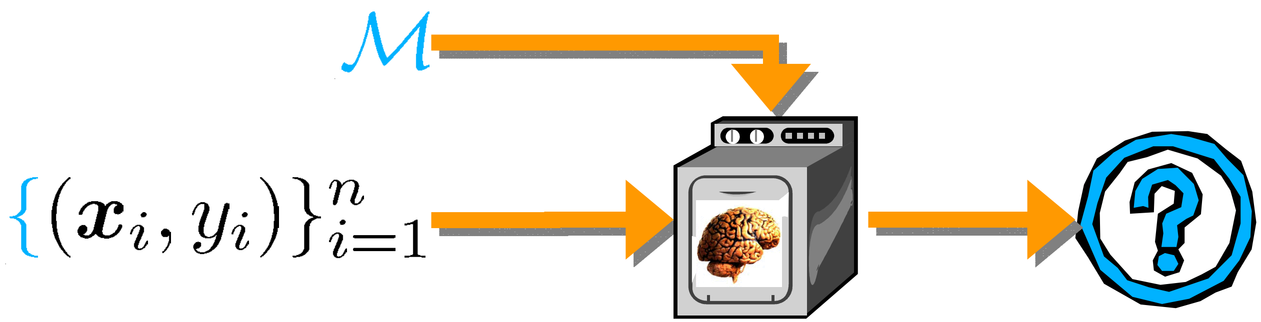


Model Selection: Analogy to Real Life

- A model represents your ambition. 
- You learn a fixed amount of material. 
- If you are less ambitious, you are not capable of even memorizing what you have learned. Therefore, you can not find the truth. 
- If you are too ambitious, you can perfectly memorize what you have learned. However, you can not get the whole picture. 
- If you are appropriately ambitious, then you can understand the truth. 

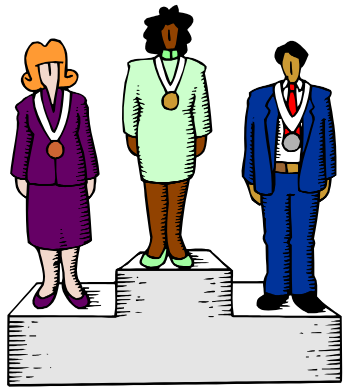
Learning Methods

- Now you have
  - A model, from which your learning result function is searched.
  - Training examples, which are pairs of questions and their answers.
- A learning method is a rule to specify a function in the model based on the training examples.



Learning Methods: Analogy to Real Life

- Now you have
  - Appropriate ambition for learning
  - Good questions and their answers
- What you should do is to just start studying!
- Effectively using your ambition and teaching materials is the key to success.



Formal Notation

- $f(\boldsymbol{x})$:Learning target function
- $\mathcal{D} \subset \mathbb{R}^d$:Domain of $f(\boldsymbol{x})$
- $\boldsymbol{x}_i$:Training input point $\boldsymbol{x}_i \overset{i.i.d.}{\sim} p(\boldsymbol{x})$
- $y_i = f(\boldsymbol{x}_i) + \epsilon_i$:Training output value
- ϵ_i :Additive random noise $\mathbb{E}_{\boldsymbol{\epsilon}} \epsilon_i = 0$
- $\{(\boldsymbol{x}_i, y_i)\}_{i=1}^n$:Training examples
- $\hat{f}(\boldsymbol{x})$:Learned function
- $\mathcal{M}$:Model

Generalization Error

- We want to obtain $\hat{f}(\boldsymbol{x})$ such that output values at unlearned test input points $\boldsymbol{t}$ can be accurately estimated.
- Suppose $\boldsymbol{t} \overset{i.i.d.}{\sim} p(\boldsymbol{x})$
- Expected test error (generalization error):

$$J = \int_{\mathcal{D}} \left(\hat{f}(\boldsymbol{t}) - f(\boldsymbol{t}) \right)^2 p(\boldsymbol{t}) d\boldsymbol{t}$$

- Goal: Obtain $\hat{f}(\boldsymbol{x})$ such that J is minimized.

Formal Description of Problems

$$J = \int_{\mathcal{D}} \left(\hat{f}(\boldsymbol{x}_{test}) - f(\boldsymbol{x}_{test}) \right)^2 p(\boldsymbol{x}_{test}) d\boldsymbol{x}$$

- Active learning: $\min_{\{\boldsymbol{x}_i\}_{i=1}^n} J$
- Model selection: $\min_{\mathcal{M}} J$
- Learning method: $\min_{\hat{f} \in \mathcal{M}} J$