Fundamental Concepts of Fracture Mechanics

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Introduction

During World War II
5000 ships built

over 1000 ships: cracks by 1946
200 ships: serious damage
9 T-2 tankers
7 Liberty ships
} broken in two

What is Fracture Mechanics?

Strength of Materials

Yield $\sigma = \sigma_y$

Failure $\sigma = \sigma_B$

Fatigue $S-N\ Diagram, Fatigue\ Limit$

Fracture Mechanics

Failure $K = K_C$ — Stress Intensity Factor K, Fracture Toughness K_C

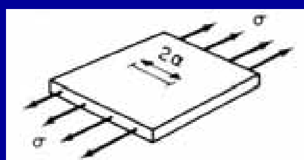
Fatigue $\frac{da}{dN} = C\left((\Delta K)^m - (\Delta K_{th})^m\right)K_C$

Fracture Mechanics is

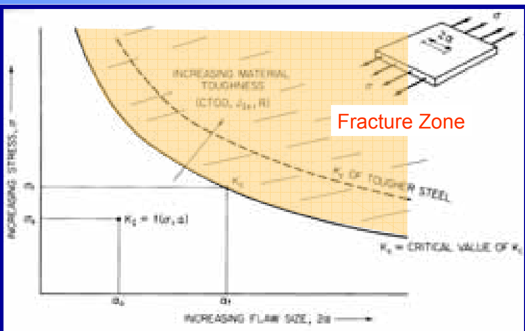
Mechanics of Members with Cracks

Stress Intensity Factor, K
Stress, σ
Crack Length, a

Fracture Toughness, Kc
Material Property



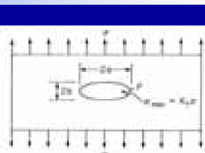
Fundamental Concepts of Fracture Mechanics



Relationship among a stress condition, a crack size and fracture toughness

Types of Discontinuities

Notches



Imperfections

Sharp Tip

Cracks

History of Fracture Mechanics

1921 Griffith

Griffith's Formula

1948 Irwin

1957 Irwin

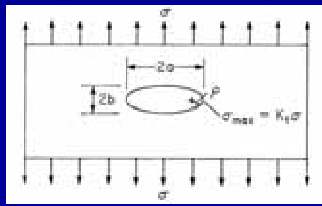
Stress Intensity Factor, K

…..

Stress Analysis of Members with Notches

Stress Concentration Factor, Kt

For An Elliptical Hole



$$K_T = \frac{\sigma_{max}}{\sigma} = 1 + \frac{2a}{b}$$

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

$$\rho = \frac{b^2}{a} \Rightarrow b = \sqrt{a\rho}$$

$$\sigma_{max} = \sigma\left(1 + 2\sqrt{\frac{a}{\rho}}\right) \Rightarrow = 2\sigma\sqrt{\frac{a}{\rho}} \; (\rho << a)$$

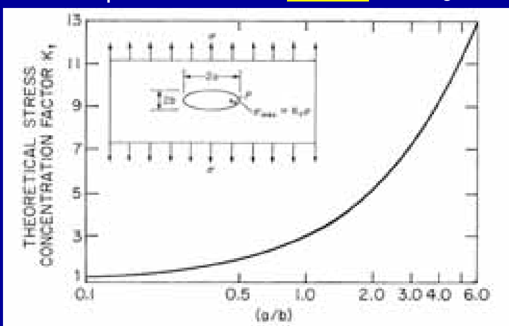
$$\sqrt{\frac{a}{\rho}} \to \infty \; (\rho \to 0)$$

$$K_t \to \infty$$

Stress Analysis of Members with Notches

Stress Concentration Factor, Kt

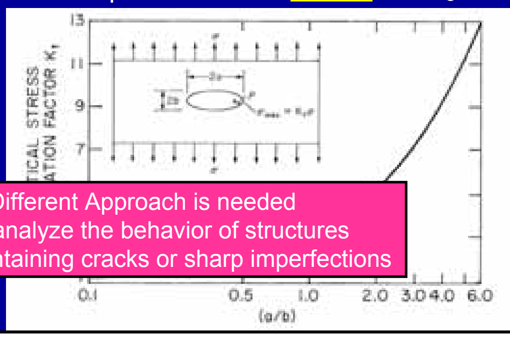
For An Elliptical Hole $K_t \to \infty$ Meaningless



Stress Analysis of Members with Notches

Stress Concentration Factor, Kt

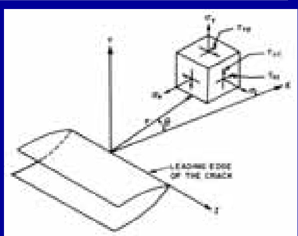
For An Elliptical Hole $K_t \to \infty$ Meaningless



A Different Approach is needed
to analyze the behavior of structures
containing cracks or sharp imperfections

Stress Analysis of Members with Notches

Stress Distribution along X-axis near the notch root



$$\frac{(\sigma_y)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r+\rho}}\left(1 + \frac{\rho}{2r+\rho}\right) + \left(\frac{\rho}{2r+\rho}\right)$$

$$0 \le r << a, \; \rho << a$$

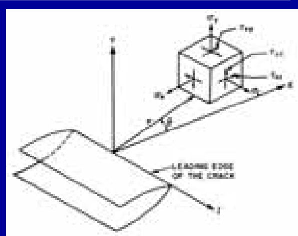
$$\frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r+\rho}}\left(1 - \frac{\rho}{2r+\rho}\right)$$

Stress Analysis of Members with Notches

Stress Distribution along X-axis near the notch root

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r+\rho}}$$

$$\rho \to 0$$



$$\frac{(\sigma_y)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r+\rho}}\left(1 + \frac{\rho}{2r+\rho}\right) + \left(\frac{\rho}{2r+\rho}\right)$$

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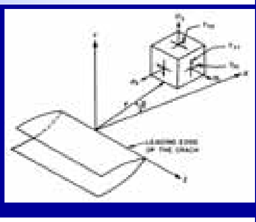
Stress Analysis of Members with Cracks

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r+\rho}}$$

$$\rho \to 0$$

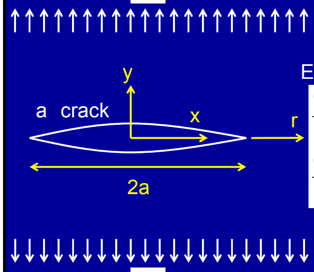


Stress near a crack tip

1. Singularity about $\frac{1}{\sqrt{r}}$
2. Intensity of Stress Singularity is Proportional to
   far field stress σ_∞
   square root of crack length a

Stress Analysis of Members with Cracks

σ_y^∞



σ_y^∞

Exact solution of stress near crack

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \frac{|x|}{\sqrt{x^2-a^2}} = \frac{a+r}{\sqrt{r(2a+r)}}$$

$$\frac{(\sigma_x)_{y=0}}{\sigma} = \frac{|x|}{\sqrt{x^2-a^2}} - 1 = \frac{a+r}{\sqrt{r(2a+r)}} - 1$$

Stress Analysis of Members with Cracks

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \frac{|x|}{\sqrt{x^2-a^2}} = \frac{a+r}{\sqrt{r(2a+r)}}$$

$$\frac{(\sigma_x)_{y=0}}{\sigma} = \frac{|x|}{\sqrt{x^2-a^2}} - 1 = \frac{a+r}{\sqrt{r(2a+r)}} - 1$$

r/a<1
Series Expansion

$$\frac{(\sigma_y)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r}} + \frac{3}{4}\sqrt{\frac{r}{2a}} - \frac{5}{32}\left(\sqrt{\frac{r}{2a}}\right)^3 + \cdots$$

$$\frac{(\sigma_x)_{y=0}}{\sigma} = \sqrt{\frac{a}{2r}} - 1 + \frac{3}{4}\sqrt{\frac{r}{2a}} - \frac{5}{32}\left(\sqrt{\frac{r}{2a}}\right)^3 + \cdots$$

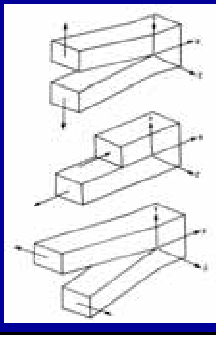
r/a>1

$$\frac{(\sigma_y)_{y=0}}{\sigma_y^\infty} \Rightarrow 1 + \frac{1}{2}\left(\frac{a}{r}\right)^2$$

$$\frac{(\sigma_x)_{y=0}}{\sigma_y^\infty} \Rightarrow \frac{1}{2}\left(\frac{a}{r}\right)^2$$

Stress Analysis for Cracks in Elastic Solids

Three Types of Relative Movements of Two Crack Surfaces



MODE I : Opening Mode

MODE II : Sliding/Shear Mode

MODE III : Tearing Mode

Basic Types of Stress Fields near Crack Tips

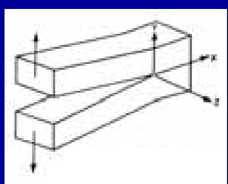
Important Basic Mode I

Most engineering situations corresponding to Mode I

$$\sigma_x = \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\cdot\sin\frac{3\theta}{2}\right]$$

$$\sigma_y = \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\cdot\sin\frac{3\theta}{2}\right]$$

$$\tau_{xy} = \frac{K_I}{(2\pi r)^{\frac{1}{2}}} \sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}\cdot\cos\frac{3\theta}{2}$$



$$u = \frac{K_I}{G}\left[\frac{r}{2\pi}\right]^{\frac{1}{2}} \cos\frac{\theta}{2}\left[1 - 2\nu + \sin^2\frac{\theta}{2}\right]$$

$$v = \frac{K_I}{G}\left[\frac{r}{2\pi}\right]^{\frac{1}{2}} \sin\frac{\theta}{2}\left[1 - 2\nu - \cos^2\frac{\theta}{2}\right]$$

$$w = 0$$

General Form of Stress Intensity Factor

The Applied Stress
The Crack Shape and Size
The Structural Configuration

⬇ Affect

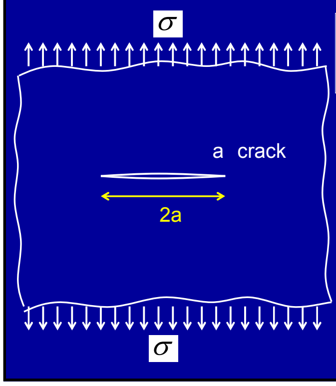
The Value of The Stress Intensity Factor, K

$$K = \sigma\sqrt{\pi a}\cdot f(g)$$

$f(g)$: Crack Geometry

Local Stress Field ⟷ Global Stress Field

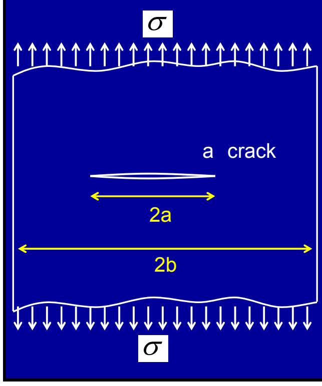
Stress Intensity Factor Equations (1)



A Through Thickness Crack
In an Infinite Plate subject to Uniform Tensile Stress

$$K = \sigma\sqrt{\pi a}$$

Stress Intensity Factor Equations (2)

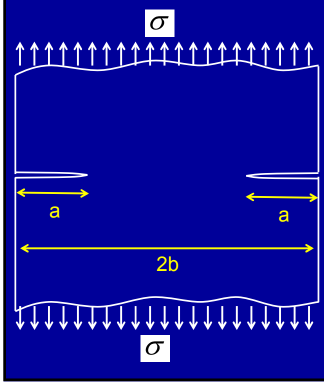


A Through Thickness Crack
In a Plate with Finite Width
Subject to
Uniform Tensile Stress

$$K = \sigma\sqrt{\pi a}\left(\frac{2b}{\pi a}\tan\frac{\pi a}{2b}\right)^{\frac{1}{2}}$$

$$= \sigma\sqrt{\pi a}\sqrt{\sec\left(\frac{\pi a}{2b}\right)}$$

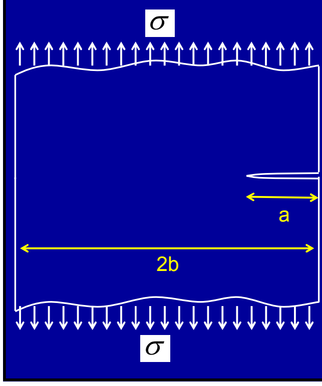
Stress Intensity Factor Equations (3)



Double-Edge Cracks
In a plate with finite width

$$K = \sigma\sqrt{\pi a}\cdot 1.12$$

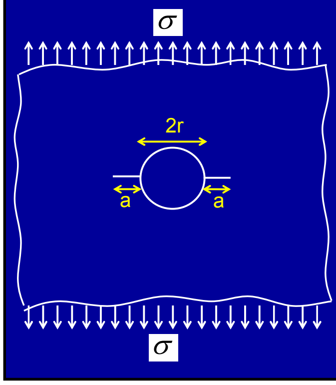
Stress Intensity Factor Equations (4)



Single-Edge Crack
In a plate with finite width

$$K = 1.12\cdot\sigma\sqrt{\pi a}\cdot f\left(\frac{a}{b}\right)$$

Stress Intensity Factor Equations (5)



Cracks growing from a round hole

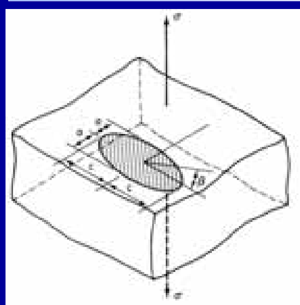
$$K = \sigma\sqrt{\pi a}\cdot f\left(\frac{a}{r}\right)$$

$$a \to 0 \quad f\left(\frac{a}{r}\right) \to 3$$

For short cracks

$$K = K_t\sigma\sqrt{\pi a}$$

Stress Intensity Factor Equations (6)



An Embedded elliptical crack or a circular crack
In an Infinite Plate

$$K = \frac{\sigma\sqrt{\pi a}}{\Phi_0}\left(\sin^2\beta + \frac{a^2}{c^2}\cos^2\beta\right)^{\frac{1}{4}}$$

$$\Phi_0 = \int_0^{\frac{\pi}{2}}\left[1 - \left(\frac{c^2 - a^2}{c^2}\right)\sin^2\theta\right]^{\frac{1}{2}} d\theta$$

$$K = \sigma\sqrt{\frac{\pi a}{Q}} \quad for \quad \beta = \frac{\pi}{2}$$

$$Q = \Phi_0^2$$

$$K = 0.65\sigma\sqrt{\pi a} = 1.15\sigma\sqrt{a}$$

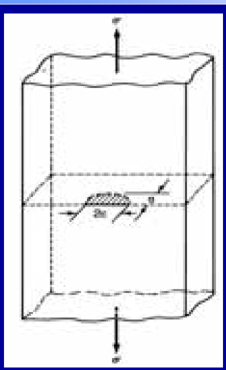
$$for \quad \beta = \frac{\pi}{2}, a = c \quad (a\ circle)$$

$$\Longleftrightarrow$$

$$K = \frac{2}{\sqrt{\pi}}\sigma\sqrt{a} = 1.13\sigma\sqrt{a}$$

Exact expression

Stress Intensity Factor Equations (6)

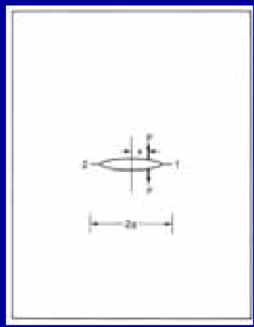


A Surface crack
or a circular crack
In an Infinite Plate

$$K = 1.12\sigma\sqrt{\pi\frac{a}{Q}}\cdot M_k$$

$$M_k = 1.0 + 1.2\left(\frac{a}{t} - 0.5\right)$$

Stress Intensity Factor Equations (7)



Cracks with wedge forces
and internal pressure

Eccentric line forces, P,
per unit thickness

$$K_{@1} = \frac{P}{\sqrt{\pi a}}\left(\frac{a+x}{a-x}\right)^{\frac{1}{2}}$$

$$K_{@2} = \frac{P}{\sqrt{\pi a}}\left(\frac{a-x}{a+x}\right)^{\frac{1}{2}}$$

$$K_{@1} = K_{@2} = \frac{P}{\sqrt{\pi a}} \quad at\ x = 0$$

$K = p\sqrt{\pi a}$: internal pressure

Engineering Calculation of Stress Intensity Factor

Pedro Albrecht/ K. Yamada, ASCE, STR, Feb.1977

$$K = F(a)\sigma\sqrt{\pi a}$$

$$F(a) = F_e \cdot F_s \cdot F_w \cdot F_g$$

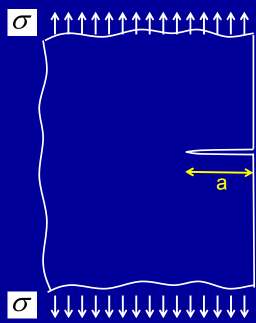
Fe: The Shape of Crack Front,
which is often assumed as an elliptical crack, correction

Fs: Effects of Free Surface, Front Free Surface Correction

Fw: Finite Width, The Back Surface Correction

Fg: Non-uniform Opening Stresses, Stress Gradient Correction

2 Dimensional Crack Problems(1)



The Edge Crack
in a semi-infinite plate

$$K = F(a)\sigma\sqrt{\pi a}$$

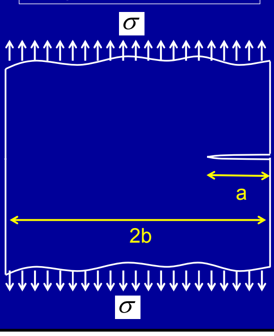
$$F(a) = F_s = 1.12$$

Front Free Surface Correction

$$K = 1.12\sigma\sqrt{\pi a}$$

2 Dimensional Crack Problems(2)

Single-Edge Crack
In a plate with finite width



$$K = F(a)\sigma\sqrt{\pi a}$$

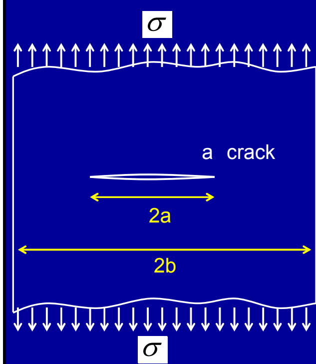
$$F(a) = F_s \cdot F_w$$

$$K = \underbrace{1.12}_{F_s}\,\sigma\sqrt{\pi a}\cdot \underbrace{f\left(\frac{a}{b}\right)}_{F_w}$$

Front Free Surface Correction

Finite Width

2 Dimensional Crack Problems (3)



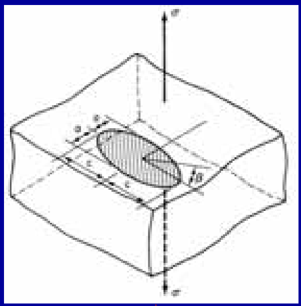
A Through Thickness Crack
In a Plate with Finite Width
Subject to
Uniform Tensile Stress

$$K = \sigma\sqrt{\pi a}\left(\frac{2b}{\pi a}\tan\frac{\pi a}{2b}\right)^{\frac{1}{2}}$$

$$= \sigma\sqrt{\pi a}\sqrt{\sec\left(\frac{\pi a}{2b}\right)}$$

Finite Width $F(a) = F_w$

3 Dimensional Crack Problems (1)



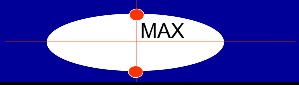
An Embedded elliptical crack
or a circular crack
In an Infinite Plate

$$K = \frac{1}{\Phi_0}\left(\sin^2\beta + \frac{a^2}{c^2}\cos^2\beta\right)^{\frac{1}{4}}\cdot\sigma\sqrt{\pi a}$$

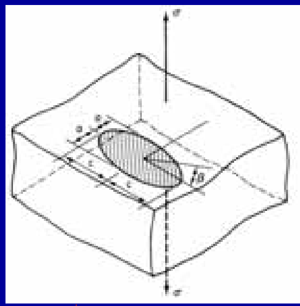
$$\Phi_0 = \int_0^{\frac{\pi}{2}}\left[1-\left(\frac{c^2-a^2}{c^2}\right)\sin^2\theta\right]^{\frac{1}{2}}d\theta$$

$$K = \boxed{\frac{1}{\Phi_0}}\sigma\sqrt{\pi a} \quad for \quad \beta = \frac{\pi}{2}$$

F_e $\quad F(a) = F_e = \frac{1}{\Phi_0}$



3 Dimensional Crack Problems (1)



An Embedded elliptical crack
or a circular crack
In an Infinite Plate

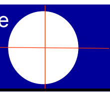
$$K = \frac{1}{\Phi_0}\left(\sin^2\beta + \frac{a^2}{c^2}\cos^2\beta\right)^{\frac{1}{4}}\cdot\sigma\sqrt{\pi a}$$

$$\Phi_0 = \int_0^{\frac{\pi}{2}}\left[1-\left(\frac{c^2-a^2}{c^2}\right)\sin^2\theta\right]^{\frac{1}{2}}d\theta$$

$$K = \boxed{\frac{1}{\Phi_0}}\sigma\sqrt{\pi a} \quad for \quad \beta = \frac{\pi}{2}$$

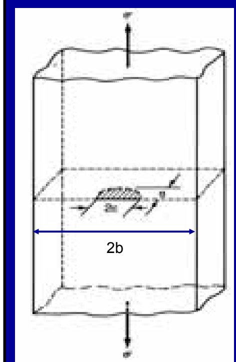
F_e $\quad F(a) = F_e = \frac{1}{\Phi_0} = \frac{2}{\pi}$ $\quad for\ a = c$

Circle



$$K = \frac{2}{\pi}\sigma\sqrt{\pi a}$$

3 Dimensional Crack Problems (2)



A Surface crack
or a circular crack
In a Plate with Finite Width

$$K = \boxed{M_k}\cdot\boxed{\frac{1}{\Phi_0}}\cdot\boxed{F_g}\,\sigma\sqrt{\pi a}\cdot\boxed{\sqrt{\frac{2b}{\pi a}\tan\frac{\pi a}{2b}}}$$

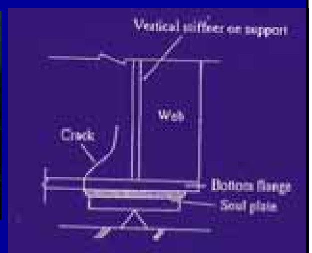
F_s $\quad F_e$ $\quad F_w$

$$M_k \begin{cases} = 1.0 + 1.2\left(\frac{a}{t} - 0.5\right) & for\ surface\ cracks \\ = 1.12 \quad for & through\ thickness\ cracks \end{cases}$$

Stress Gradient Correction
Correction for Stress Concentration

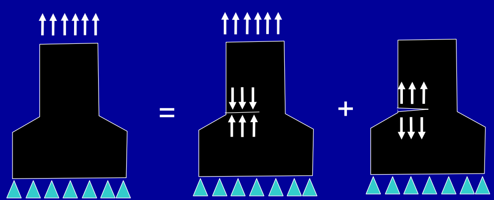
Geometry Correction Factor F_g

Cracks always occur at geometrical discontinuities as cover plate ends …..



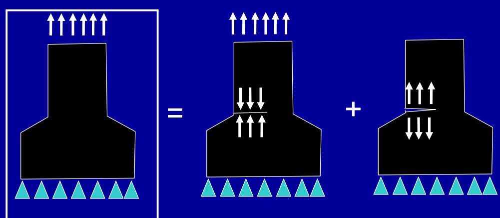
Calculation of F_g

Superimpose



Calculation of F_g

Superimpose

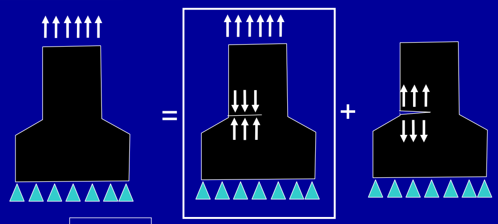


Step 1

Compute the actual stress
Along the line where the crack
Shall be inserted FEM
By any suitable methods.

Calculation of F_g

Superimpose

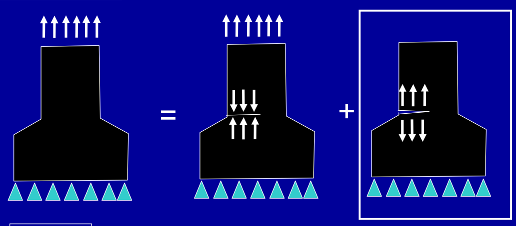


Step 2

Insert a crack of given length
Along the same line

Calculation of F_g

Superimpose

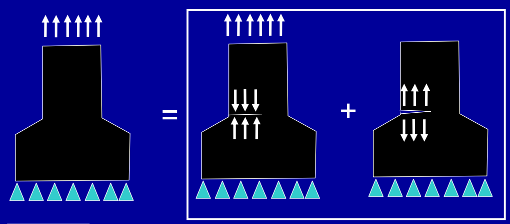


Step 3

Compute K by integrating away the normal, determined in Step 1 stresses applied over the length of the crack

Calculation of F_g

Superimpose



Step 4

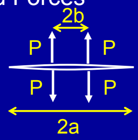
Repeat steps 2 and 3
for any desired crack size

Calculation of F_g

Calculation of K in Step 3

Concentrated Forces

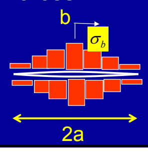
a crack



$$K = \frac{2P}{\sqrt{\pi a}} \frac{a}{\sqrt{a^2 - b^2}}$$

Distributed Forces

a crack



$$K = \sqrt{\pi a}\,\frac{2}{\pi}\int_0^a \frac{\sigma_b}{\sqrt{a^2 - b^2}}\,db$$

Calculation of F_g

Stress Distribution in Step 1

Closed form expressions defining the stress distribution
In the uncracked body are usually not available.

From FEM

$$K = \sqrt{\pi a}\,\frac{2}{\pi}\sum_{i=1}^{n} \sigma_{b_i} \int_{b_i}^{b_{i-1}} \frac{1}{\sqrt{a^2 - b^2}}\,db$$

In which the discrete stress σ_{b_i} is applied
over the element b_i and b_{i+1}

Calculation of F_g

Stress Distribution in Step 1

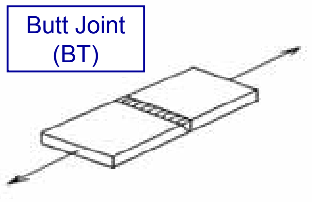
After factoring out the mean stress

$$K = \sqrt{\pi a}\,\frac{2}{\pi}\sum_{i=1}^{n} \sigma_{b_i} \int_{b_i}^{b_{i-1}} \frac{1}{\sqrt{a^2 - b^2}}\,db$$

$$K = \sigma\sqrt{\pi a}\,\frac{2}{\pi}\sum_{i=1}^{n} \frac{\sigma_{b_i}}{\sigma}\left(\arcsin\frac{b_{i-1}}{a} - \arcsin\frac{b_i}{a}\right)$$

Examples of F_g

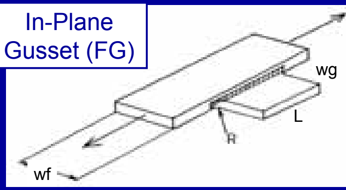
Butt Joint (BT)



$$F_g = 1$$

Examples of F_g

In-Plane Gusset (FG)

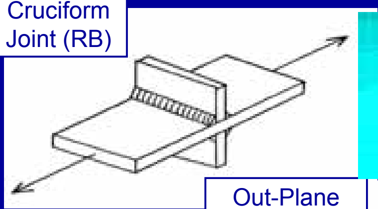


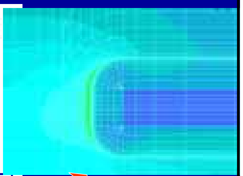
$$F_g = \frac{-1.115\log(\frac{R}{w_f}) + 0.537\log(\frac{L}{w_f}) + 0.1384\log(\frac{w_g}{w_f}) + 0.6801}{1 + \frac{1}{1.158}\cdot(\frac{a}{w_f})^{0.6051}}$$

By Zettlemoyer and Fisher

Examples of F_g

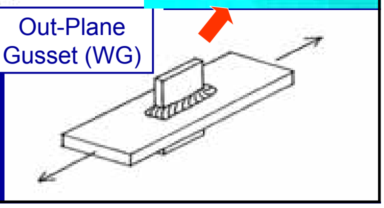
Cruciform Joint (RB)

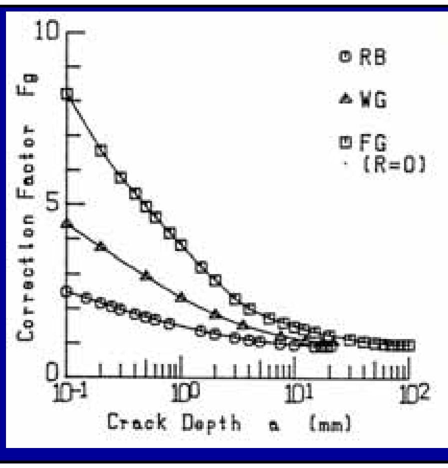




Stress Distribution From FEM

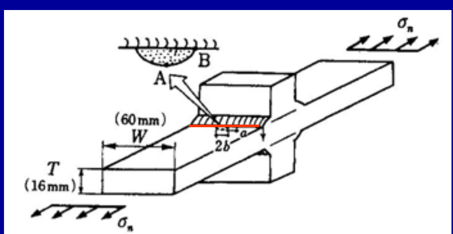
Out-Plane Gusset (WG)



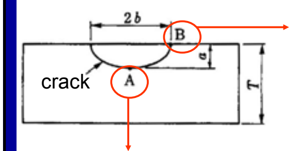


Examples of F_g

Cracks from Incomplete Penetration



Model of Cracks



$$\Delta K_B = F_{eB}\cdot F_{tB}\cdot F_{gB}\cdot\Delta\sigma\cdot\sqrt{\pi a}$$

$$F_{eB} = F_{eA}\Big/\sqrt{a/b}$$

$$F_{tB} = (1-0.025k^2+0.06k^4)\sqrt{\sec(\pi k/2)}$$

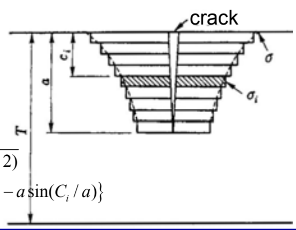
$$\Delta K_A = F_{eA}\cdot F_{sA}\cdot F_{tA}\cdot F_{gA}\cdot\Delta\sigma\cdot\sqrt{\pi a}$$

$$F_{eA} = 1\Big/\sqrt{1+1.464(a/b)^{1.65}}$$

$$F_{sA} = 1+0.12(1-a/b)$$

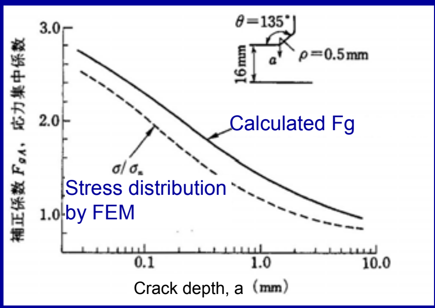
$$F_{tA} = (1-0.025\lambda^2+0.06\lambda^4)\sqrt{\sec(\pi\lambda/2)}$$

$$F_{gA} = (2/\pi)\sum(\sigma_i/\sigma_n)\{a\sin(C_{i+1}/a) - a\sin(C_i/a)\}$$

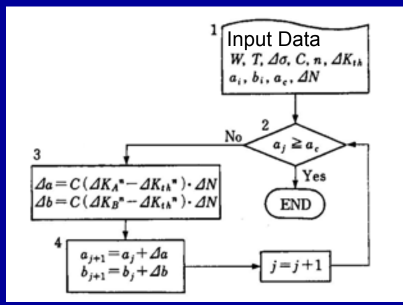


Calculation of Fg

Relationships of stress distribution and Fg

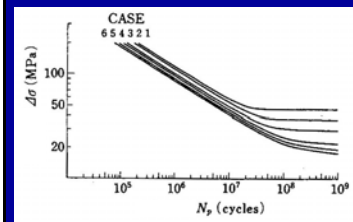


Calculation strategies



Results

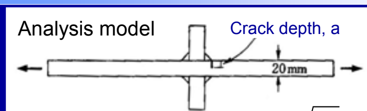
Variation of initial crack width and height



Initial crack geometry

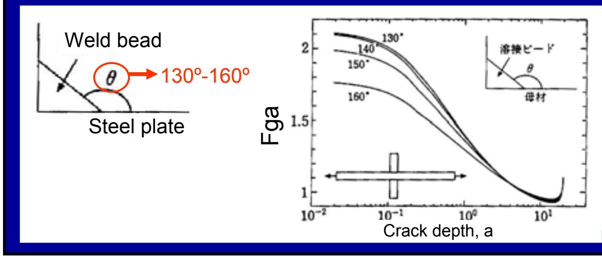
CASE	depth a (mm)	width $2b$ (mm)
1	0.1	0.4
2	0.2	0.8
3	0.5	2.0
4	1.0	4.0
5	1.5	6.0
6	2.0	8.0

Effects of weld shape

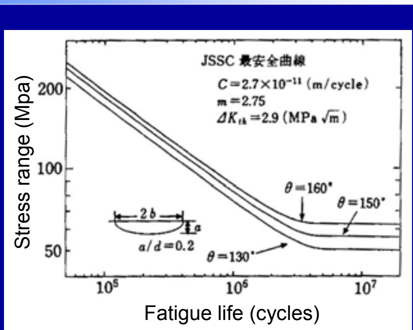


a_o = 0.1 mm
$2b_o$ = 1.0 mm

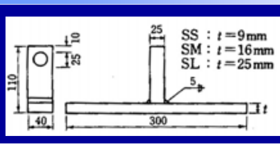
$$\Delta K_A = F_{eA} \cdot F_{sA} \cdot F_{tA} \cdot F_{gA} \cdot \Delta\sigma \cdot \sqrt{\pi a}$$



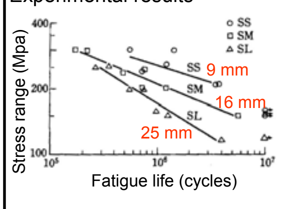
Effects of weld shape



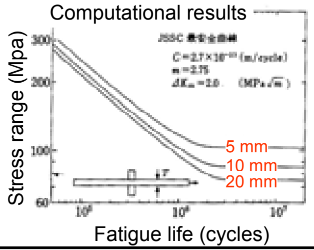
Effects of plate thickness



Experimental results



Computational results



Fracture Toughness

The Condition for Unstable Crack Growth

$$K = K_c$$

Fracture Toughness

The Critical Stress Intensity Factor for Unstable Crack Growth

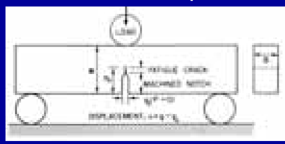
Mode I

$$K_I = K_{Ic}$$

Test Methods to Evaluate K_{Ic}

ASTM E-399

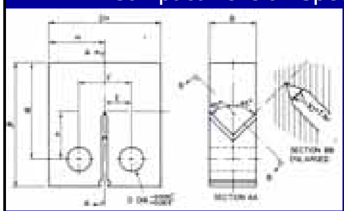
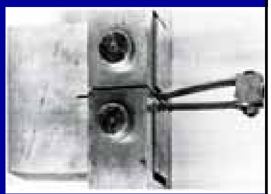
A Bend Specimen



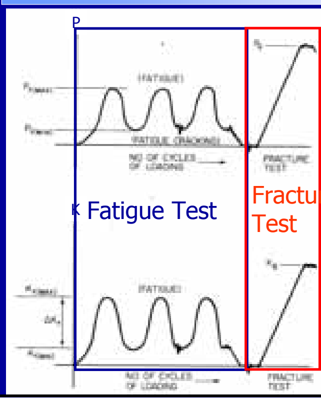

Test Methods to Evaluate K_{Ic}

ASTM E-399

Compact Tension Specimen

Test Methods to Evaluate K_{Ic}



The Testing Program

Fatigue Cracking

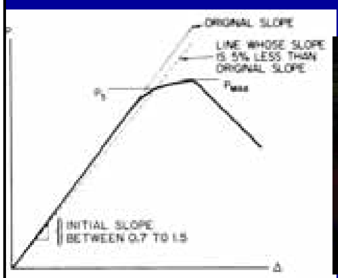
Small Scale Yielding Condition

Fracture Test

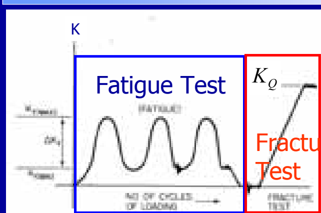
Plane Strain Condition

Test Methods to Evaluate K_{Ic}

Fracture Tests



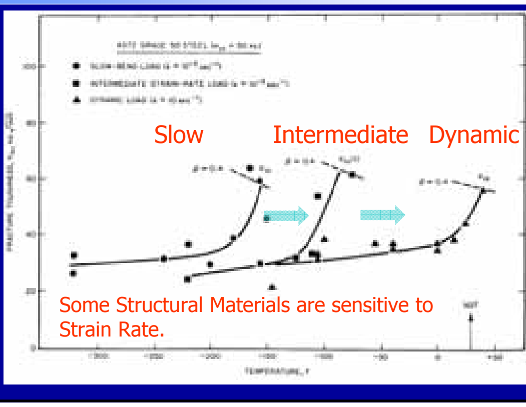
Test Methods to Evaluate K_{Ic}



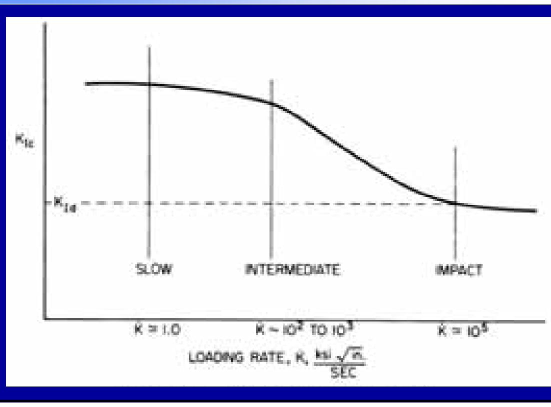
$$a \geq 2.5\left(\frac{K_Q}{\sigma_{Yield}}\right)^2, B \geq 2.5\left(\frac{K_Q}{\sigma_{Yield}}\right)^2, W \geq 5.0\left(\frac{K_Q}{\sigma_{Yield}}\right)^2$$

$$K_Q \Rightarrow K_{Ic}$$

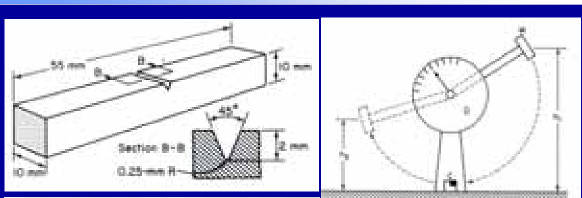
Effects of Loading Rate on Fracture Toughness



Effects of Loading Rate on Fracture Toughness



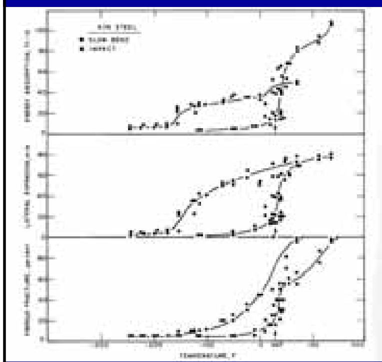
Charpy Impact Tests



Charpy V-Notch (CVN) Impact Tests

The Traditional Evaluation Method of Fracture Toughness

Charpy Impact Tests



To Prevent Brittle Fracture

Absorbed Energy: Ev

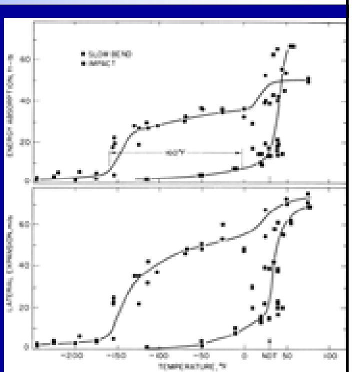
Transition Temperature: Ttr

Absorbed Energy-Temperature

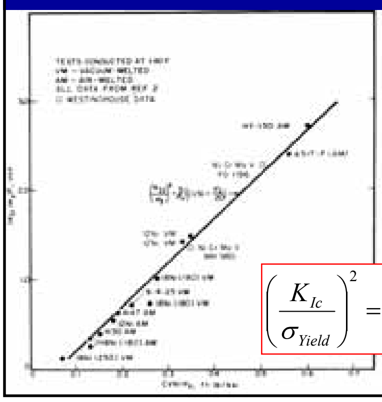
CVN Tests vs K1c Tests

CVN Tests:
Dynamic Condition
Crack Initiation
+ Propagation

K1c Tests:
Slow Bending
Crack Initiation



K1c- CVN Upper Shelf Correlation



Proposed by Barsom and Rolf

$$\left(\frac{K_{Ic}}{\sigma_{Yield}}\right)^2 = 5\left(\frac{CVN}{\sigma_{Yield}} - 0.05\right)$$