2.Transmission Line

2-1.Transmission-line equations

$$\frac{dV(z)}{dz} = -Z_d I(z) \qquad (Z_d = R + j\omega L)$$

$$\frac{dI(z)}{dz} = -Y_d V(z) \qquad (Y_d = G + j\omega C)$$

$$V = V_1 e^{-\gamma z} + V_2 e^{+\gamma z} = V_i e^{+\gamma y} + V_r e^{-\gamma y}$$

$$I = \frac{1}{Z_c}(V_1 e^{-\gamma z} - V_2 e^{+\gamma z}) = \frac{1}{Z_c}(V_i e^{+\gamma y} - V_r e^{-\gamma y})$$

where propagation constant and characteristic impedance are given by

$$\gamma = \sqrt{Z_d Y_d} = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

$$\frac{Z_d}{\gamma} = \sqrt{\frac{Z_d}{Y_d}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = Z_c$$

2-2.Reflection coefficient and input impedance

Since the load impedance is given by

$$Z_L = \frac{V(0)}{I(0)} = Z_c \frac{V_i + V_r}{V_i - V_r}$$

the reflection coefficient is defined by

$$\frac{V_r}{V_i} = \frac{Z_L - Z_c}{Z_L + Z_c} = S_v(0)$$

The impedance observed at an arbitrary position is given by

$$Z(y) = Z_c \frac{Z_L + Z_c \tanh \gamma y}{Z_c + Z_L \tanh \gamma y}$$

In case of loss-less transmission line, the impedance is given by

$$Z(y) = R_c \frac{Z_L + jR_c \tan \beta y}{R_c + jZ_L \tan \beta y}$$

2-3. Standing wave ratio

$$\rho = \frac{|V(y)|_{\max}}{|V(y)|_{\min}} = \frac{1+|S(0)|}{1-|S(0)|}$$

2-4. λ/4 impedance transformer

The following relation holds in a loss-less transmission line.

$$z(l+\frac{\lambda}{4}) = \frac{z_L + j\tan\beta(l+\frac{\lambda}{4})}{1+jz_L\tan\beta(l+\frac{\lambda}{4})} = \frac{jz_L\tan\beta l + 1}{j\tan\beta l + z_L} = \frac{1}{z(l)}$$

$$Z(l+\frac{\lambda}{4})Z(l) = R_c^2$$

The load resistance R_L can be matched to the line with the characteristic impedance R_c by using a λ/4 segment of transmission line with a characteristic impedance

$$R_x = \sqrt{R_c R_L}$$

2-5. Smith chart