物質：対流拡散方程式

$$\frac{\partial c}{\partial t}+v_x\frac{\partial c}{\partial x}+v_y\frac{\partial c}{\partial y}+v_z\frac{\partial c}{\partial z}=-\left(\frac{\partial \Phi_x}{\partial x}+\frac{\partial \Phi_y}{\partial y}+\frac{\partial \Phi_z}{\partial z}\right)-R \qquad \text{(A)}$$

特徴：

- $c=C$ [mol・m^{-3}] ：**物質の濃度**
- **分子効果による移動：** (A)で $\Phi_x=-D\dfrac{\partial C}{\partial x}$
- **消失**R：**反応速度(消失を正にとる)**

$$\frac{\partial C}{\partial t}+v_x\frac{\partial C}{\partial x}+v_y\frac{\partial C}{\partial y}+v_z\frac{\partial C}{\partial z}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right)-R$$

熱：熱移動方程式

$$\frac{\partial c}{\partial t}+v_x\frac{\partial c}{\partial x}+v_y\frac{\partial c}{\partial y}+v_z\frac{\partial c}{\partial z}=-\left(\frac{\partial \Phi_x}{\partial x}+\frac{\partial \Phi_y}{\partial y}+\frac{\partial \Phi_z}{\partial z}\right)-R$$

特徴：

- $c=\rho C_p T$ [J・m^{-3}]
- **分子効果による移動：**

  (A)で $\Phi_x=-\dfrac{k}{\rho C_p}\dfrac{\partial \rho C_p T}{\partial x}=-\alpha\dfrac{\partial \rho C_p T}{\partial x}$
- **消失**R：**熱損失**

  (摩擦による損失,外部からの仕事等)

$$\frac{\partial \rho C_p T}{\partial t}+v_x\frac{\partial \rho C_p T}{\partial x}+v_y\frac{\partial \rho C_p T}{\partial y}+v_z\frac{\partial \rho C_p T}{\partial z}=$$

$$\frac{k}{\rho C_p}\left(\frac{\partial^2 \rho C_p T}{\partial x^2}+\frac{\partial^2 \rho C_p T}{\partial y^2}+\frac{\partial^2 \rho C_p T}{\partial z^2}\right)-R$$

↓ ρ，C_p一定

$$\rho C_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)-R$$

対流の流束：$\rho C_p T v_x$

熱拡散流束(分子効果)：$\Phi_x = q_x = -\alpha\dfrac{\partial \rho C_p T}{\partial x}$　α：**熱拡散係数**

Fourier**の熱伝導の法則**：$q_x = -k\dfrac{\partial T}{\partial x}$　k[J・m^{-1}・K^{-1}・s^{-1}] (k:**熱伝導度**)

$\Phi_x = -\kappa\dfrac{\partial c}{\partial x}$　[X・m^{-2}・s^{-1}]　　c [X・m^{-3}]，κ [m^2・s^{-1}]

T[K] ⟶ $\rho C_p T$ [J・m^{-3}]

k[J・m^{-1}・K^{-1}・s^{-1}] ⟶ $\dfrac{k}{\rho C_p}=\alpha$ [m$^2\cdot$s^{-1}]

運動量：運動方程式

運動量収支とは？何で運動量収支？

運動方程式：$F=ma$

$$F = ma = m\frac{dv}{dt}=\frac{d(mv)}{dt}$$

運動量（mv）

↓

力=**運動量の時間変化**

運動量収支式　――　**運動方程式**

$$\frac{\partial c}{\partial t}+v_x\frac{\partial c}{\partial x}+v_y\frac{\partial c}{\partial y}+v_z\frac{\partial c}{\partial z}=-\left(\frac{\partial \Phi_x}{\partial x}+\frac{\partial \Phi_y}{\partial y}+\frac{\partial \Phi_z}{\partial z}\right)-R \qquad \text{(A)}$$

特徴：

- $c=\rho \boldsymbol{v}$ [kg・m・s^{-1}・m^{-3}] **ベクトル**
- **分子効果による移動：**(A)**で**$\Phi=\tau$ **テンソル**
- **圧力重力による生成：**$-R$ [(kg・m・s^{-1})・m^{-3}・s^{-1}]

- $\rho \boldsymbol{v}$ **ベクトル：**3**成分についての式**
  (**以下ではとりあえず**x**成分だけ**)

- **分子効果による移動：** $\Phi=\tau$

$$[\mathrm{N\cdot m^{-2}}]\rightarrow[\mathrm{kg\cdot m\cdot s^{-2}\cdot m^{-2}}]\rightarrow[(\mathrm{kg\cdot m\cdot s^{-1}})\cdot \mathrm{m^{-2}}\cdot \mathrm{s^{-1}}]$$

3**つの成分×**3**つの方向**=9**成分**

$$\Phi=\tau=\begin{pmatrix}\tau_{xx} & \tau_{yx} & \tau_{zx}\\ \tau_{xy} & \tau_{yy} & \tau_{zy}\\ \tau_{xz} & \tau_{yz} & \tau_{zz}\end{pmatrix}$$

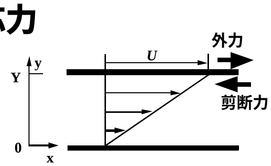
剪断応力　法線応力

$\tau_{xy}=\tau_{yx}$　$\tau_{xx},\tau_{yy},\tau_{zz}$

$\tau_{yz}=\tau_{zy}$

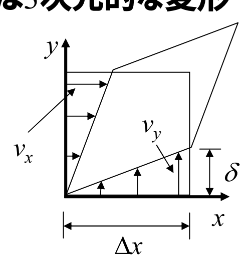
$\tau_{zx}=\tau_{xz}$

剪断応力



$$\tau_{yx}=-\mu\frac{du}{dy}$$

実際は3**次元的な変形**



$$\text{変形率}=\frac{\delta}{\Delta x}=\frac{\partial v_y}{\partial x}\ [\mathrm{s^{-1}}]$$

2つの変形を合わせた変形速度： $\dot{\gamma} = \dfrac{\partial v_y}{\partial x} + \dfrac{\partial v_x}{\partial y}$ [s^{-1}]

$$\tau_{xy} = \tau_{yx} = \mu \dot{\gamma}_{xy} = \mu \left(\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right)$$

法線応力

$$\tau_{xx} = -p + \mu \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_x}{\partial x} \right) + \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

$$= -p + 2\mu \frac{\partial v_x}{\partial x} + \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

$$\tau_{xx} = -p + 2\mu \frac{\partial v_x}{\partial x} + \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

$$\tau_{yy} = -p + 2\mu \frac{\partial v_y}{\partial y} + \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

$$\tau_{zz} = -p + 2\mu \frac{\partial v_z}{\partial z} + \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

$$p = -\frac{1}{3}\left(\tau_{xx} + \tau_{yy} + \tau_{zz} \right)$$

$$= -p + \frac{2}{3}\mu \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right) + \lambda \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$$

↓

$$\lambda = -\frac{2}{3}\mu$$

$$\frac{\partial c}{\partial t}+v_x\frac{\partial c}{\partial x}+v_y\frac{\partial c}{\partial y}+v_z\frac{\partial c}{\partial z}=-\left(\frac{\partial \Phi_x}{\partial x}+\frac{\partial \Phi_y}{\partial y}+\frac{\partial \Phi_z}{\partial z}\right)-R$$

3次元：とりあえずx成分

- $c=\rho v_x$
- $\Phi_y=\tau_{yx}=-\mu\left(\frac{\partial v_y}{\partial x}+\frac{\partial v_x}{\partial y}\right)$
- $\Phi_x=\tau_{xx}=-2\mu\frac{\partial v_x}{\partial x}$

圧力項は生成の中に入れたことと

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0$$ **であるため**

- $-R=-\frac{\partial p}{\partial x}+\rho g_x$

$$\frac{\partial \rho v_x}{\partial t}+v_x\frac{\partial \rho v_x}{\partial x}+v_y\frac{\partial \rho v_x}{\partial y}+v_z\frac{\partial \rho v_x}{\partial z}=2\mu\frac{\partial^2 v_x}{\partial x^2}$$

$$+\mu\frac{\partial}{\partial y}\left(\frac{\partial v_y}{\partial x}+\frac{\partial v_x}{\partial y}\right)+\mu\frac{\partial}{\partial z}\left(\frac{\partial v_x}{\partial z}+\frac{\partial v_z}{\partial x}\right)-\frac{\partial p}{\partial x}+\rho g_x$$

↓ $\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0$

$$\frac{\partial \rho v_x}{\partial t}+v_x\frac{\partial \rho v_x}{\partial x}+v_y\frac{\partial \rho v_x}{\partial y}+v_z\frac{\partial \rho v_x}{\partial z}=-\frac{\partial p}{\partial x}$$

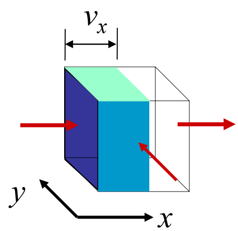
$$+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

↓

$$\rho\frac{Dv_x}{Dt}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

$$\rho \frac{Dv_x}{Dt} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) + \rho g_x$$

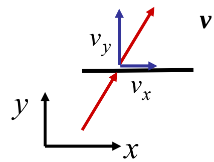
運動量のx方向成分の流束



対流

x**方向**: $\rho v_x v_x$

y**方向**: $\rho v_x v_y$

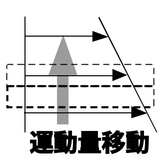


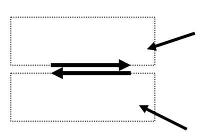
対流流束：テンソル

3つの成分×3つの方向=9成分

$$\begin{pmatrix} \rho v_x v_x & \rho v_x v_y & \rho v_x v_z \\ \rho v_y v_x & \rho v_y v_y & \rho v_y v_z \\ \rho v_z v_x & \rho v_z v_y & \rho v_z v_z \end{pmatrix}$$

運動量流束（分子効果）：応力がなぜ運動量流束？





下層が上層に
及ぼす剪断応力

上が下に及ぼす応力

力=運動量の時間変化

単位時間に

上層：図の応力×面積の分だけ運動量が増加

下層：同じだけ運動量が減少

↓

下層から上層に運動量が移動したように見える → **粘性による運動量移動**

$$\rho\frac{Dv_x}{Dt}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)+\rho g_x$$

$$\rho\frac{Dv_y}{Dt}=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)+\rho g_y$$

$$\rho\frac{Dv_z}{Dt}=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)+\rho g_z$$

↓

ベクトル表示 $\mathbf{v}=v_x\mathbf{i}+v_y\mathbf{j}+v_z\mathbf{k}$

$$\rho\frac{Dv_x}{Dt}\boldsymbol{i}=-\frac{\partial p}{\partial x}\boldsymbol{i}+\mu\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)\boldsymbol{i}+\rho g_x\boldsymbol{i}$$

$$\rho\frac{Dv_y}{Dt}\boldsymbol{j}=-\frac{\partial p}{\partial y}\boldsymbol{j}+\mu\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}+\frac{\partial^2 v_y}{\partial z^2}\right)\boldsymbol{j}+\rho g_y\boldsymbol{j}$$

$$\rho\frac{Dv_z}{Dt}\boldsymbol{k}=-\frac{\partial p}{\partial z}\boldsymbol{k}+\mu\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial y^2}+\frac{\partial^2 v_z}{\partial z^2}\right)\boldsymbol{k}+\rho g_z\boldsymbol{k}$$

ベクトル演算子の導入

ナブラ： $\nabla=\frac{\partial}{\partial x}\boldsymbol{i}+\frac{\partial}{\partial y}\boldsymbol{j}+\frac{\partial}{\partial z}\boldsymbol{k}$

ラプラシアン： $\nabla^2=\nabla\cdot\nabla=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$

$$\frac{Dv_x}{Dt}\boldsymbol{i}+\frac{Dv_y}{Dt}\boldsymbol{j}+\frac{Dv_z}{Dt}\boldsymbol{k}=\frac{D\boldsymbol{v}}{Dt}$$

$$-\left(\frac{\partial p}{\partial x}\boldsymbol{i}+\frac{\partial p}{\partial y}\boldsymbol{j}+\frac{\partial p}{\partial z}\boldsymbol{k}\right)=-\nabla p=-\mathrm{grad}p$$

$$\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}+\frac{\partial^2 v_x}{\partial z^2}\right)\boldsymbol{i}=\nabla^2 v_x\boldsymbol{i}$$

$$\nabla^2 v_x\boldsymbol{i}+\nabla^2 v_y\boldsymbol{j}+\nabla^2 v_z\boldsymbol{k}=\nabla^2\boldsymbol{v}$$

↓

$$\rho \frac{D\boldsymbol{v}}{Dt} = -\nabla p + \mu \nabla^2 \boldsymbol{v} + \rho \boldsymbol{g}$$

↓

$$\nabla \cdot \boldsymbol{\tau} = \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) \boldsymbol{i} + \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) \boldsymbol{j} + \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \right) \boldsymbol{k}$$

$$\rho \frac{D\boldsymbol{v}}{Dt} = -\nabla p - \nabla \cdot \tau + \rho \boldsymbol{g}$$