移動現象操作

数値シミュレーション

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1.基礎方程式:非圧縮性流体

連続の式 $\mathrm{div}\,\boldsymbol{v} = 0$

運動方程式

$$\frac{\partial \boldsymbol{v}}{\partial \boldsymbol{t}} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\frac{1}{\rho}\mathrm{grad}p + \nu\Delta\boldsymbol{v} + \boldsymbol{K}$$

$\boldsymbol{K}$:外力 (流体単位質量あたり)
重力:g、ローレンツ力:$\boldsymbol{j} \times \boldsymbol{B}$

$\boldsymbol{j}$:電流密度, $\boldsymbol{B}$:磁束密度

連続の式 $$\frac{\partial \rho}{\partial t} + \mathrm{div}\rho \boldsymbol{v} = 0$$

運動方程式

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v}) = -\mathrm{grad}p - \nabla \cdot \tau + \rho \boldsymbol{K}$$

エネルギー方程式

$$\rho \frac{DE_t}{Dt} = -\mathrm{div}(-k\mathrm{grad}T) - \mathrm{div}\rho \boldsymbol{v} + \rho \boldsymbol{v} \cdot \boldsymbol{K} + \mathrm{div}\, \tau \cdot \boldsymbol{v} + \rho Q$$

基礎方程式の特徴

非圧縮性流体

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) + K$$

1)**二階微分の項にかかるパラメータ**

$$\nu \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right) \xrightarrow{\text{無次元化}} \frac{1}{\mathrm{Re}} \left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right)$$

2) **非線形の項の存在**

$$v_x \frac{\partial v_x}{\partial x}$$

1)**二階の項のパラメータ**

$$\frac{du}{dx}=\frac{1}{\mathrm{Re}}\frac{d^2u}{dx^2}\quad(0<x<1)$$

$$u(0)=1,\ u(1)=0$$

$$\rightarrow u=\frac{\exp(\mathrm{Re})-\exp(\mathrm{Re}\,x)}{\exp(\mathrm{Re})-1}$$

境界付近で大きな変化

2)**非線形項**

- **解の重ね合わせができない**
- **ひとつの特殊解から他の特殊解を類推しにくい**

$$v_x=u(t)\sin kx \longrightarrow v_x\frac{\partial v_x}{\partial x}$$

K**:波数**

$$\downarrow$$

$$v_x\frac{\partial v_x}{\partial x}=u(t)\sin kx\cos kx=\frac{1}{2}u(t)\sin 2kx$$

高波数成分が現れる

2.差分法の基礎

離散化の方法･留意すべき点

差分化:微分方程式を代数方程式で近似

例：関数Tの微分を差分化 $(x_{i+1} = x_i + \Delta x)$

$$T(x_{i+1}) = T(x_i) + \Delta x \frac{dT}{dx} + \frac{\Delta x^2}{2!}\frac{d^2T}{dx^2} + \frac{\Delta x^3}{3!}\frac{d^3T}{dx^3} + \cdots$$

$$\frac{dT}{dx} = \frac{T(x_{i+1}) - T(x_i)}{\Delta x} \underline{- \frac{\Delta x^2}{2!}\frac{d^2T}{dx^2} - \frac{\Delta x^3}{3!}\frac{d^3T}{dx^3} - \cdots}$$

$$\frac{dT}{dx} \approx \frac{T(x_{i+1}) - T(x_i)}{\Delta x}$$

これらの項を無視

$$T(x_{i+1}) = T(x_i) + \Delta x \frac{dT}{dx} + \frac{\Delta x^2}{2!}\frac{d^2T}{dx^2} + \frac{\Delta x^3}{3!}\frac{d^3T}{dx^3} + \cdots$$

$$T(x_{i-1}) = T(x_i) - \Delta x \frac{dT}{dx} + \frac{\Delta x^2}{2!}\frac{d^2T}{dx^2} - \frac{\Delta x^3}{3!}\frac{d^3T}{dx^3} + \cdots$$

↓

$$\frac{dT}{dx} = \frac{T(x_{i+1}) - T(x_{i-1})}{2\Delta x} - \frac{2\Delta x^3}{3!}\frac{d^3T}{dx^3} - \cdots$$

$$\frac{dT}{dx} \approx \frac{T(x_{i+1}) - T(x_{i-1})}{2\Delta x}$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0 \quad (0<x<1, 0<y<1)$$

$$T(x,0)=0,\quad T(x,1)=0,\quad T(0,y)=0,\quad T(1,y)=1$$

差分化

$$\frac{d^2 T}{dx^2}=aT_{i-1}+bT_i+cT_{i+1}$$
$$=aT(x_{i-1})+bT(x_i)+cT(x_{i+1})$$
$$=aT(x_i-\Delta x)+bT(x_i)+cT(x_i+\Delta x)$$

$$aT(x_i-\Delta x)=a(T(x_i)-\Delta xT'(x_i)+\frac{1}{2}\Delta x^2 T''(x_i)+\cdots)$$

$$cT(x_i+\Delta x)=c(T(x_i)+\Delta xT'(x_i)+\frac{1}{2}\Delta x^2 T''(x_i)+\cdots)$$

$$\downarrow$$

$$aT(x_{i-1})+bT(x_i)+cT(x_{i+1})=(a+b+c)T(x_i)+(a-c)\Delta xT'(x_i)$$
$$(a+c)\frac{1}{2}\Delta x^2 T''(x_i)+\cdots$$

$$\downarrow$$

$$a+b+c=0,\quad a-c=0,\quad (a+c)\frac{1}{2}\Delta x^2=1$$

$$a=c=\frac{1}{\Delta x^2},\quad b=-\frac{2}{\Delta x^2}$$

$$\frac{d^2T}{dx^2} \approx aT(x_{i-1}) + bT(x_i) + cT(x_{i+1}) = \frac{T(x_{i-1}) - 2T(x_i) + T(x_{i+1})}{\Delta x^2}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

↓

$$\frac{T(x_{i+1}, y_i) - 2T(x_i, y_i) + T(x_{i-1}, y_i)}{\Delta x^2} + \frac{T(x_i, y_{i+1}) - 2T(x_i, y_i) + T(x_i, y_{i-1})}{\Delta y^2} = 0$$

内部の格子上の点全てについて成り立つ

↓

格子の数の一次方程式を連立して解く

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}, \quad (0 < x < 1, t > 0)$$

$$T(0,t) = T(1,t) = 0, \quad T(x,0) = f(x)$$

↓ **差分化**

$$\frac{T^{n+1}(x_i) - T^n(x_i)}{\Delta t} = \frac{T^n(x_{i+1}) - 2T^n(x_i) + T^n(x_{i-1})}{\Delta x^2}$$

← **前進差分**

↓

$$T^{n+1}(x_i) = rT^n(x_{i+1}) + (1-2r)T^n(x_i) + rT^n(x_{i-1}), \quad r = \frac{\Delta t}{\Delta x^2}$$

$$\frac{T^n(x_i)-T^{n-1}(x_i)}{\Delta t}=\frac{T^n(x_{i+1})-2T^n(x_i)+T^n(x_{i-1})}{\Delta x^2}$$

後退差分

$$-rT^n(x_{i-1})+(1+2r)T^n(x_i)-rT^n(x_{i+1})=T^{n-1}(x_i)$$:**連立方程式**

$$T^{n+1}(x_i)=rT^n(x_{i+1})+(1-2r)T^n(x_i)+rT^n(x_{i-1})$$:**陽解法**

$$-rT^n(x_{i-1})+(1+2r)T^n(x_i)-rT^n(x_{i+1})=T^{n-1}(x_i)$$:**陰解法**

3.数値解析の方法

渦度法

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}\right)$$

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}\right)$$

↓

$$\frac{\partial \omega}{\partial t}+v_x\frac{\partial \omega}{\partial x}+v_y\frac{\partial \omega}{\partial y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \omega}{\partial x^2}+\frac{\partial^2 \omega}{\partial y^2}\right)$$ **渦度輸送方程式**

流れ関数･渦度法

$$\frac{\partial \omega}{\partial t}+v_x\frac{\partial \omega}{\partial x}+v_y\frac{\partial \omega}{\partial y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right)$$

連続の式

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=0$$

流れ関数：ψ　$v_x=\frac{\partial \psi}{\partial y},\ v_y=-\frac{\partial \psi}{\partial x}$

$$\frac{\partial \omega}{\partial t}+\frac{\partial \psi}{\partial y}\frac{\partial \omega}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial \omega}{\partial y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right)$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-\omega$$

境界条件：$\omega,\ \psi$

MAC法：圧力の取り扱い

$$\frac{\partial v_x}{\partial t}+v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial y^2}\right)$$　xで微分

$$\frac{\partial v_y}{\partial t}+v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_y}{\partial x^2}+\frac{\partial^2 v_y}{\partial y^2}\right)$$　yで微分

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=D$$

$$\frac{\partial D}{\partial t}+\frac{\partial}{\partial x}\left(v_x\frac{\partial v_x}{\partial x}+v_y\frac{\partial v_x}{\partial y}\right)+\frac{\partial}{\partial y}\left(v_x\frac{\partial v_y}{\partial x}+v_y\frac{\partial v_y}{\partial y}\right)=-\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}\right)+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 D}{\partial x^2}+\frac{\partial^2 D}{\partial y^2}\right)$$

$$\left(\frac{\partial v_x}{\partial x}\right)^2+2\frac{\partial v_x}{\partial y}\frac{\partial v_y}{\partial x}+\left(\frac{\partial v_y}{\partial y}\right)^2+v_x\frac{\partial D}{\partial x}+v_y\frac{\partial D}{\partial y}$$

$$\frac{\partial D}{\partial t}\approx\frac{D^{n+1}-D^n}{t},\ D^{n+1}=0$$

MAC法

$$\frac{\partial^2 p^n}{\partial x^2}+\frac{\partial^2 p^n}{\partial y^2}=-\left\{\left(\frac{\partial v_x^n}{\partial x}\right)^2+2\frac{\partial v_x^n}{\partial y}\frac{\partial v_y^n}{\partial x}+\left(\frac{\partial v_y^n}{\partial y}\right)^2\right\}+\frac{D^n}{\Delta t}$$

$$-v_x^n\frac{\partial D^n}{\partial x}-v_y^n\frac{\partial D^n}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 D^n}{\partial x^2}+\frac{\partial^2 D^n}{\partial y^2}\right)$$

$$v_x^{n+1}=v_x^n+\Delta t\left\{-v_x^n\frac{\partial v_x^n}{\partial x}-v_y^n\frac{\partial v_x^n}{\partial y}-\frac{\partial p^n}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_x^n}{\partial x^2}+\frac{\partial^2 v_x^n}{\partial y^2}\right)\right\}$$

$$v_y^{n+1}=v_y^n+\Delta t\left\{-v_x^n\frac{\partial v_y^n}{\partial x}-v_y^n\frac{\partial v_y^n}{\partial y}-\frac{\partial p^n}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_y^n}{\partial x^2}+\frac{\partial^2 v_y^n}{\partial y^2}\right)\right\}$$

スタッガード格子を使用